Disc temperature structure

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Outline

- Review of radiation processes
- Disc heating / cooling processes
- A simple model for disc temperature structure

Emission and absorption of radiation

- Kirchhoff's law:

$$j_\nu(T) = \kappa_\nu(T) B_\nu(T)$$

Emissivity

Opacity

Planck's function

The ratio of emissivity and opacity is a universal function of frequency and temperature only.

Emission and absorption of radiation

- Kirchhoff's law:

$$j_\nu(T) = \kappa_\nu(T) B_\nu(T)$$

- Planck-mean opacity:

$$\kappa_P(T) = \frac{\int_0^\infty \kappa_\nu(T) B_\nu(T) d\nu}{\int_0^\infty B_\nu(T) d\nu} = \frac{\int_0^\infty \kappa_\nu(T) B_\nu(T) d\nu}{\sigma T^4 / \pi}$$

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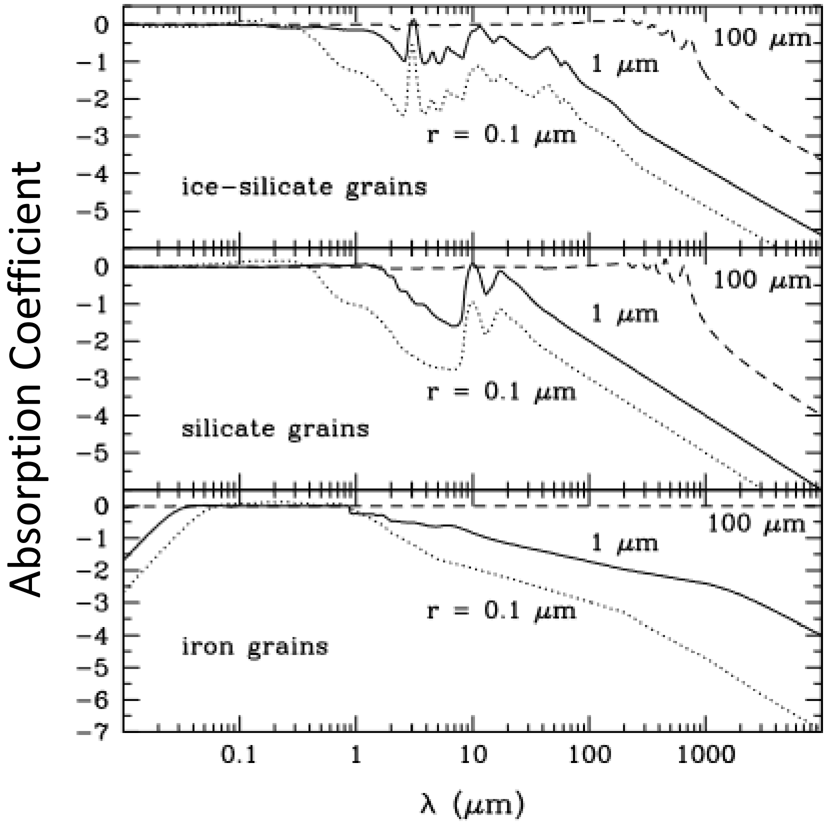
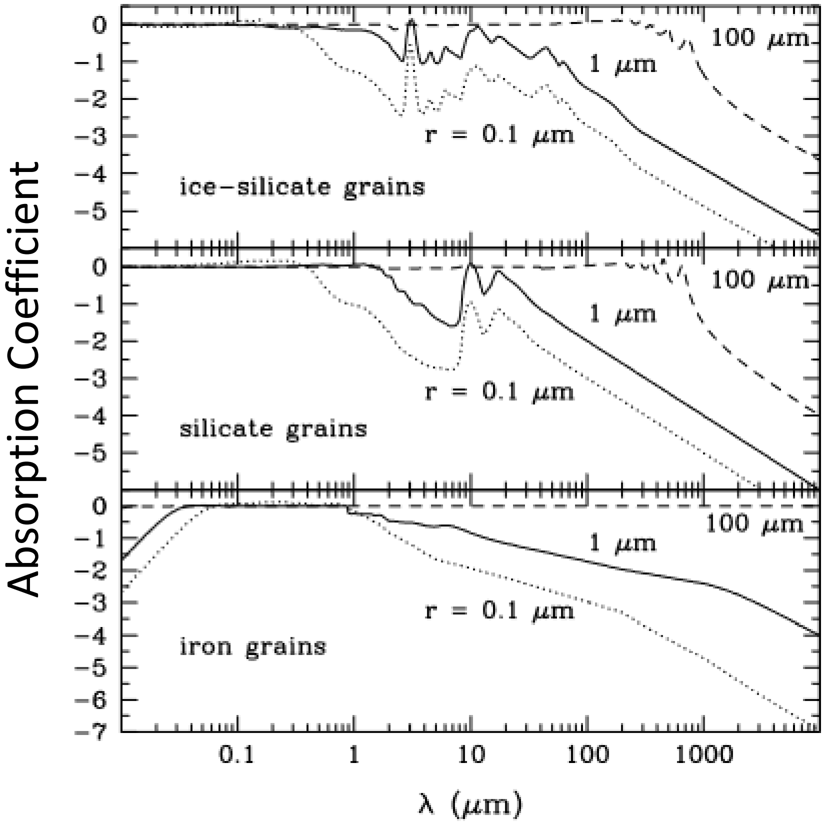
- Kirchhoff's law implies $\kappa_P(T)$ tells us:
  - The energy absorbed from a black-body radiation field of temperature, T
  - The energy emitted at temperature T.

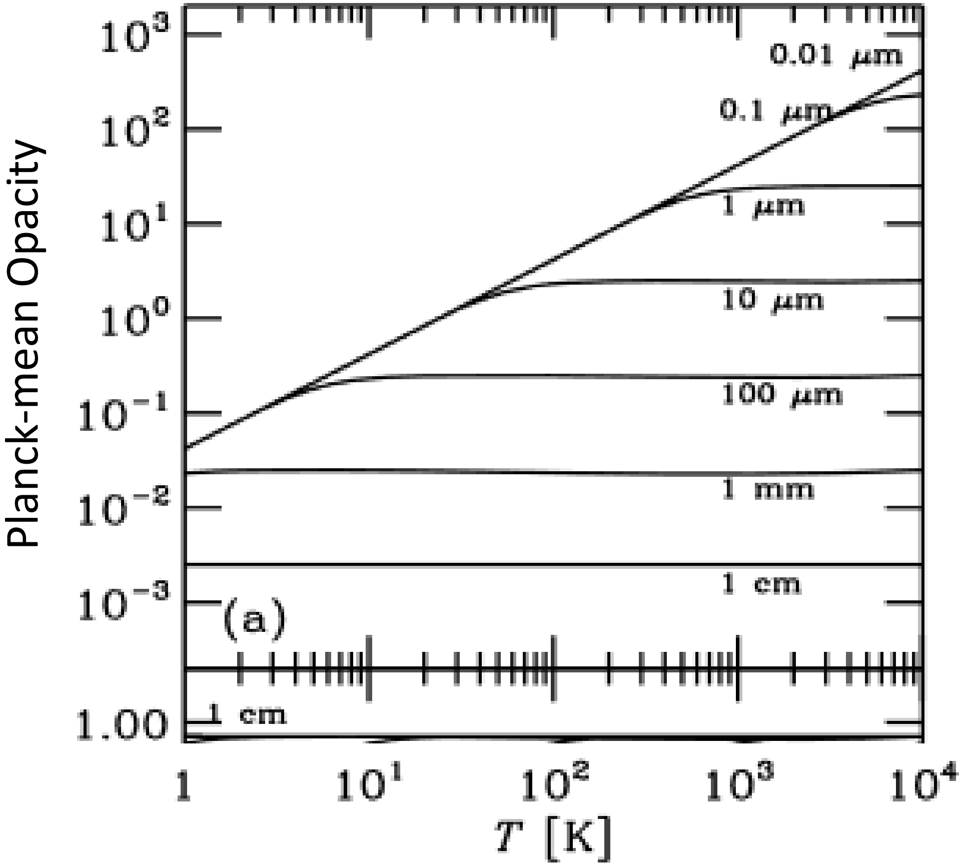
Dust Opacity



Dust is the main continuum opacity source in PPDs

Dust Opacity

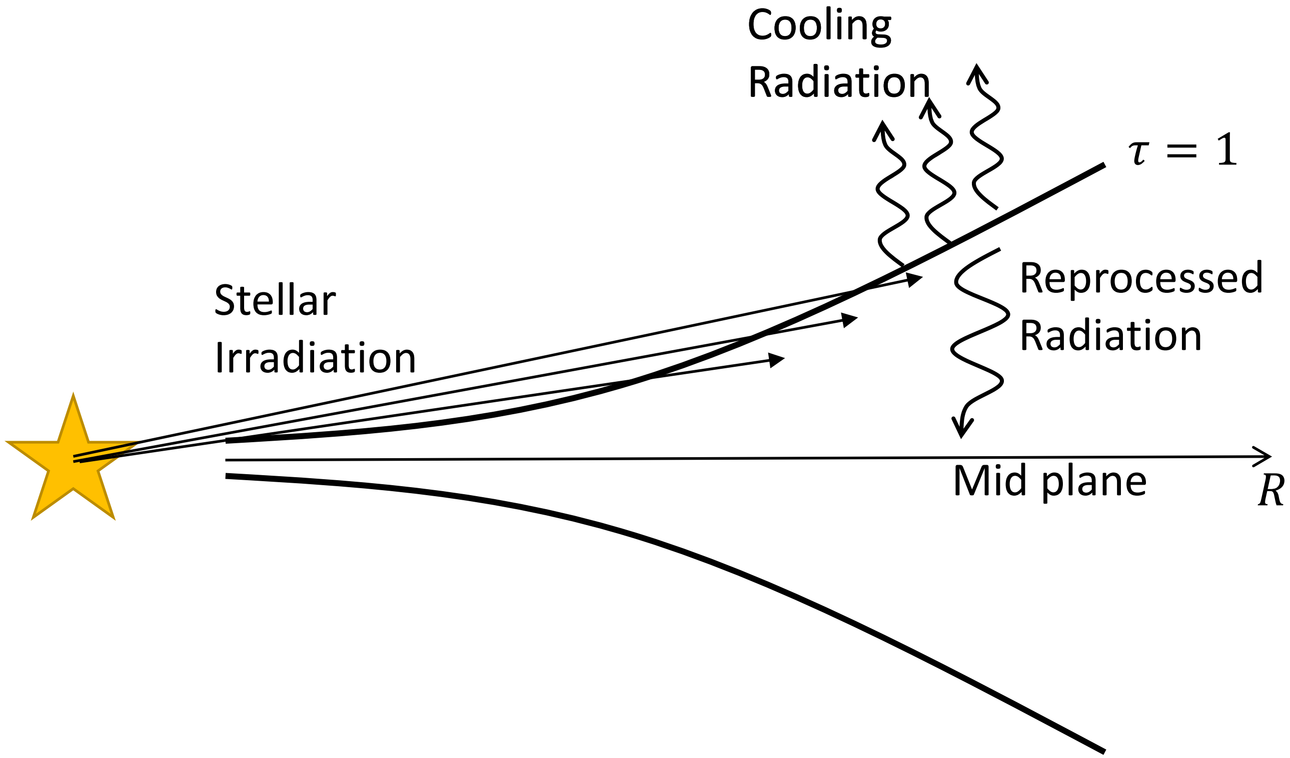




Dust is the main continuum opacity source in PPDs

Heating and Cooling

Thermal Radiation



Heating and Cooling

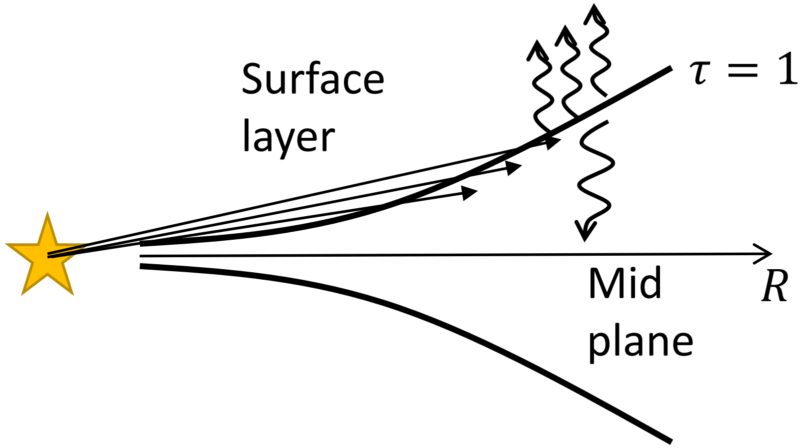
Surface layer temperature

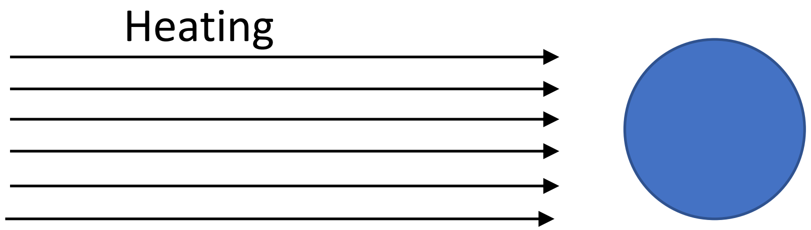
- Heated directly by the star
- Cools radiatively.

Assume black body grain:

Heating rate:

$$F_* \, \pi a^2$$





Heating and Cooling

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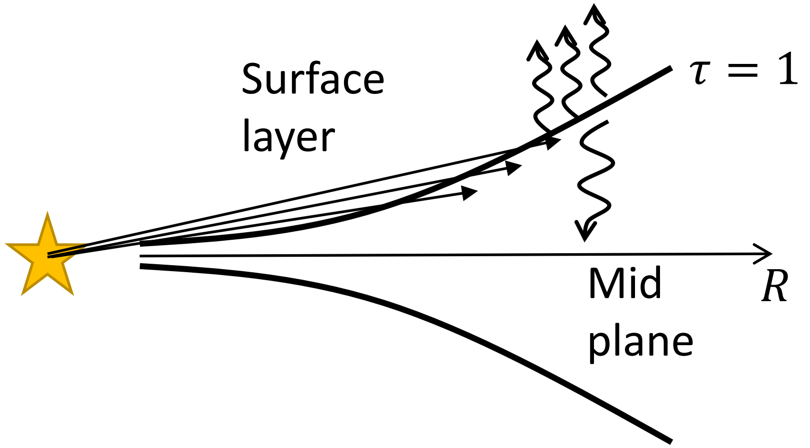
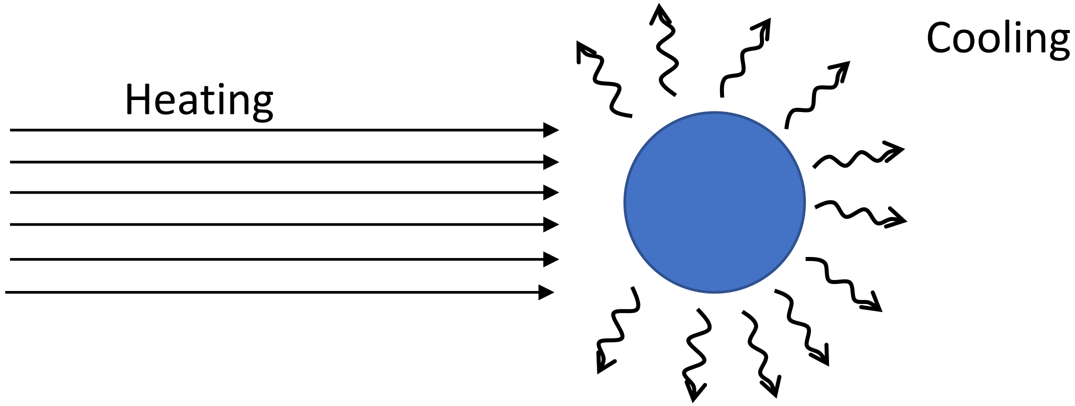
Cooling rate:

$$4\pi a^2 \, \sigma T^4$$





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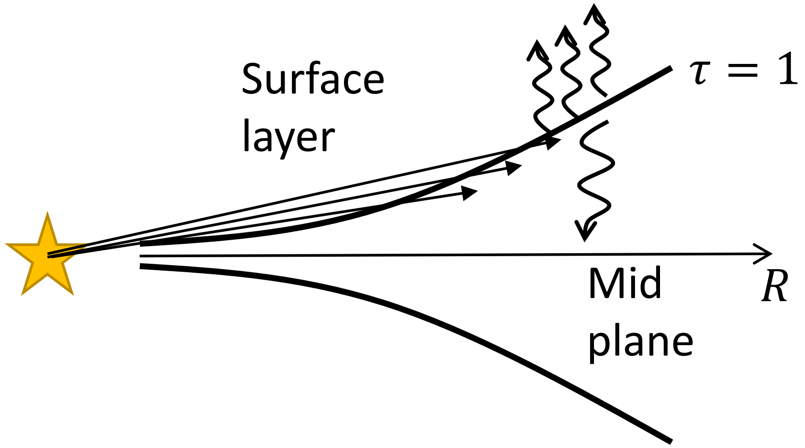
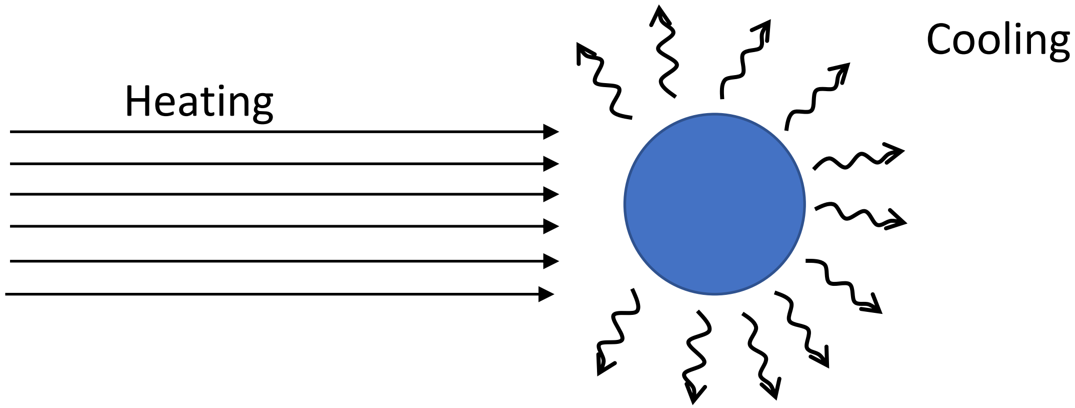
Stellar flux:

$$F_* = \frac{L}{4\pi R^2} = \frac{4\pi R_*^2 \sigma T_*^4}{4\pi R^2} = \sigma \left(\frac{R_*}{R}\right)^2 T_*^4$$





Heating and Cooling

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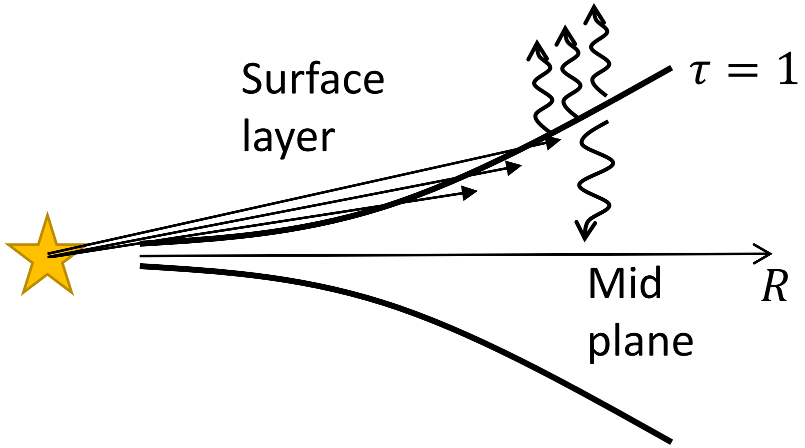

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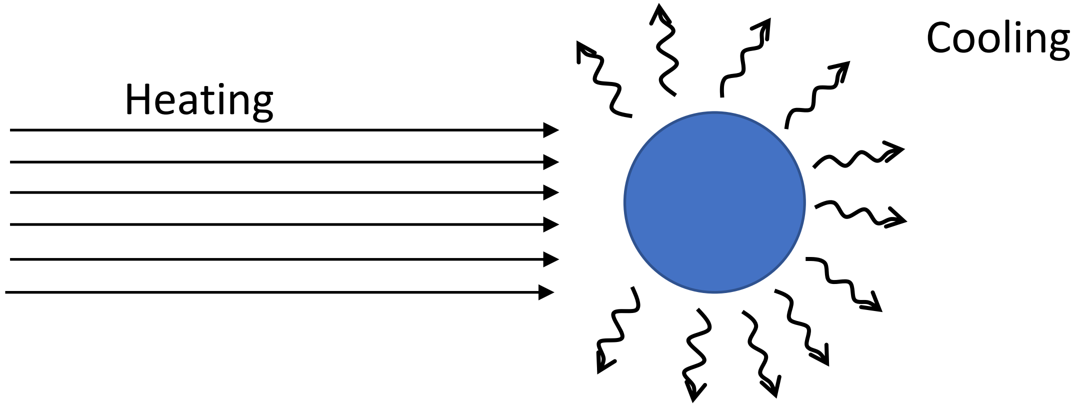
Dust temperature (surface):

$$T_s = \left(\frac{R_*}{2R}\right)^{\frac{1}{2}} T_*$$



Heating and Cooling
Surface layer temperature

- Large (black body grains)

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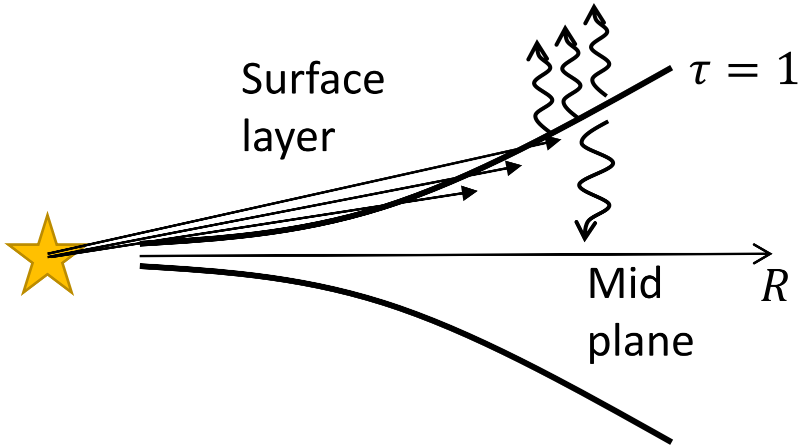
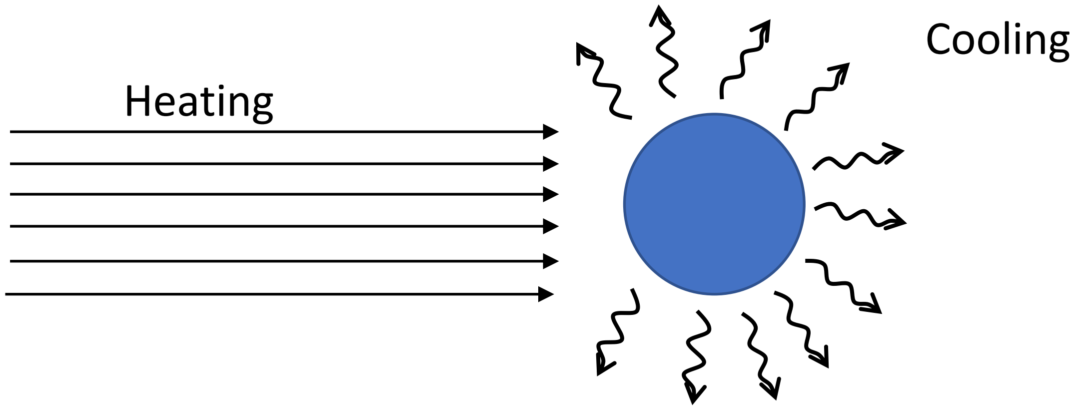




Heating and Cooling

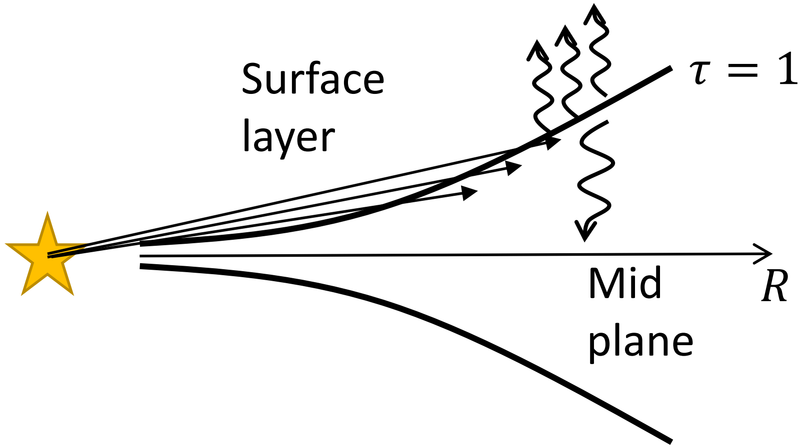
Surface layer temperature

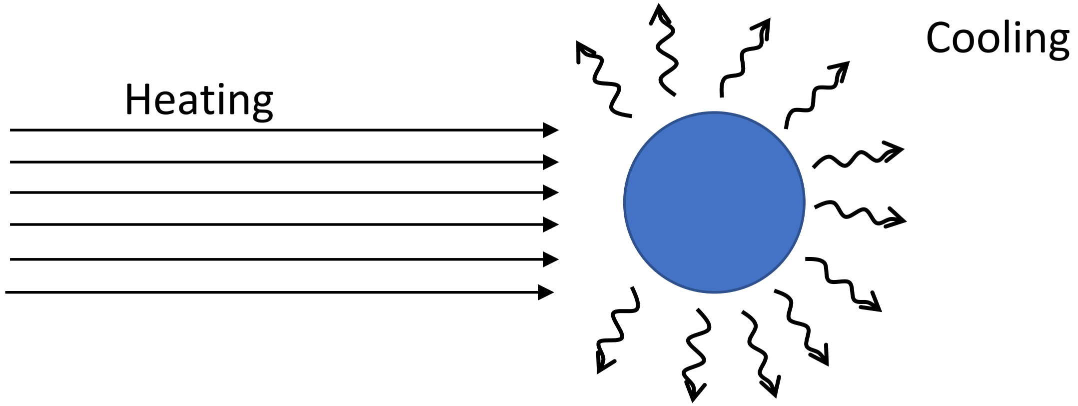
- Large (black body grains)

$$T_s = \left(\frac{R_*}{2R}\right)^{\frac{1}{2}} T_*$$

- But, if the grain is small then

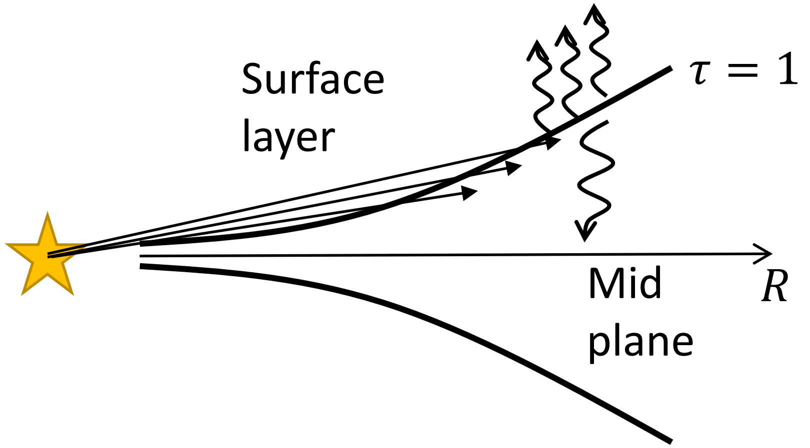
$$\kappa_P(T_*) \neq \kappa_P(T_d)$$





Heating and Cooling

Surface layer temperature



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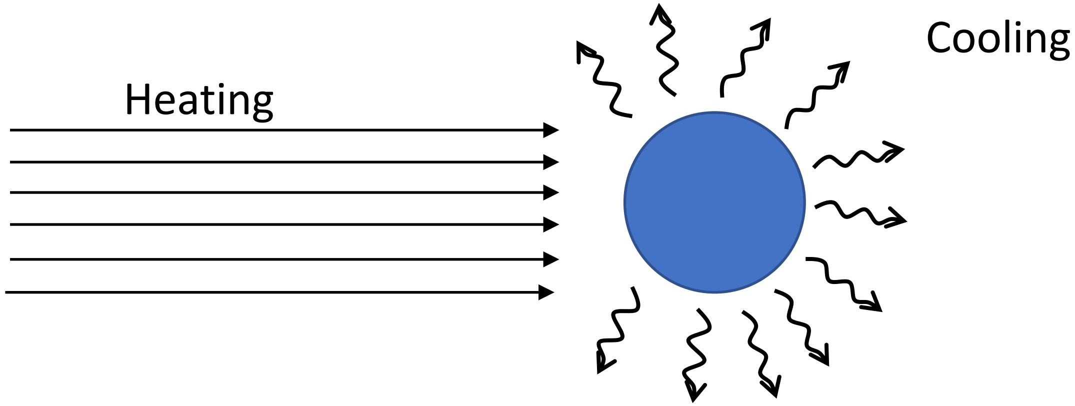
- But, if the grain is small then

$$\kappa_P(T_*) \neq \kappa_P(T_s)$$

Hence

$$\left(\frac{R_*}{R}\right)^2 \kappa_P(T_*)\, \sigma T_*^4 = 4\kappa_P(T_s)\sigma T_s^4$$



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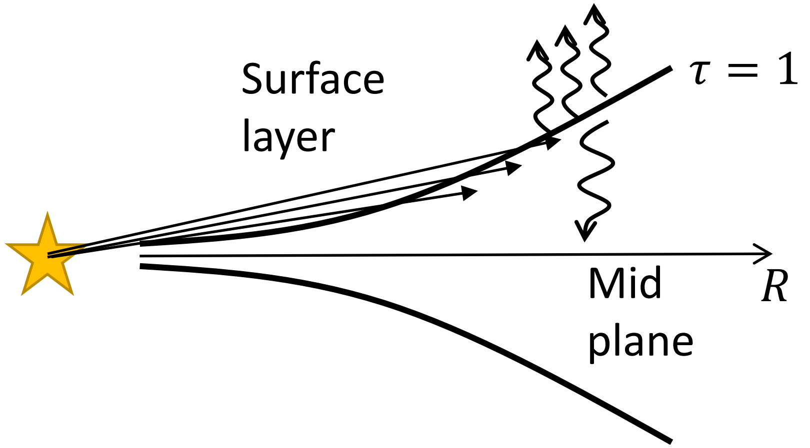
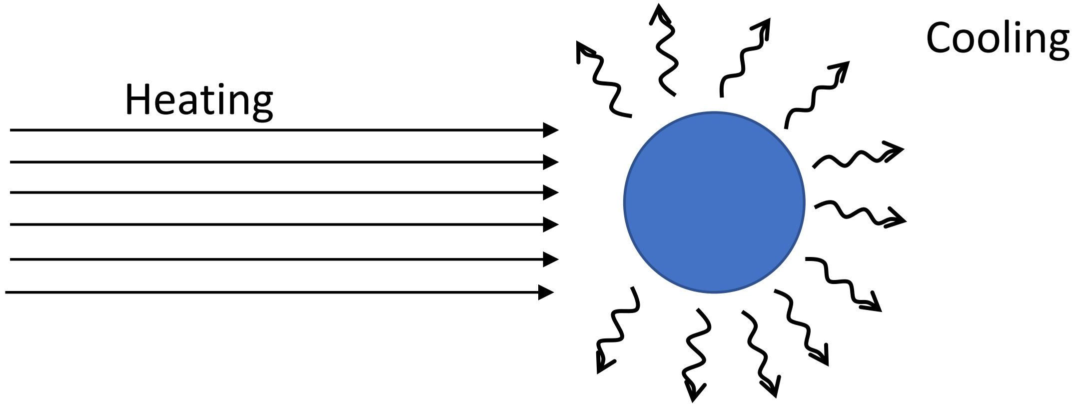


Hence

$$\left(\frac{R_*}{R}\right)^2 \kappa_P(T_*)\,\sigma T_*^4 = 4\kappa_P(T_s)\sigma T_s^4$$

$$\boxed{T_s = \left(\frac{\kappa_P(T_*)}{\kappa_P(T_s)}\right)^{\frac{1}{4}} \left(\frac{R_*}{2R}\right)^{\frac{1}{2}} T_*}$$



Note: We ignored the radiation from the disc's interior when computing this temperature

Heating and Cooling

Interior temperature

- *Flux* impinging on the disc surface *per unit area:*

$$F_* \sin(\alpha) \approx \alpha \left(\frac{R_*}{R}\right)^2 \sigma T_*^4$$

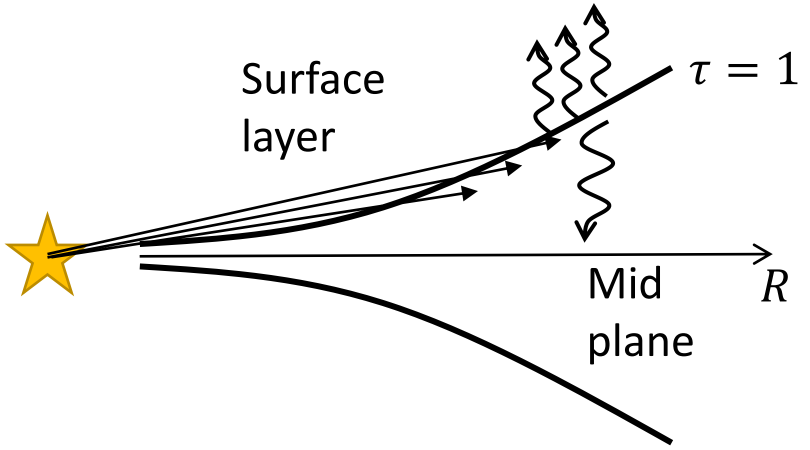
Half of the flux is emitted up/down.

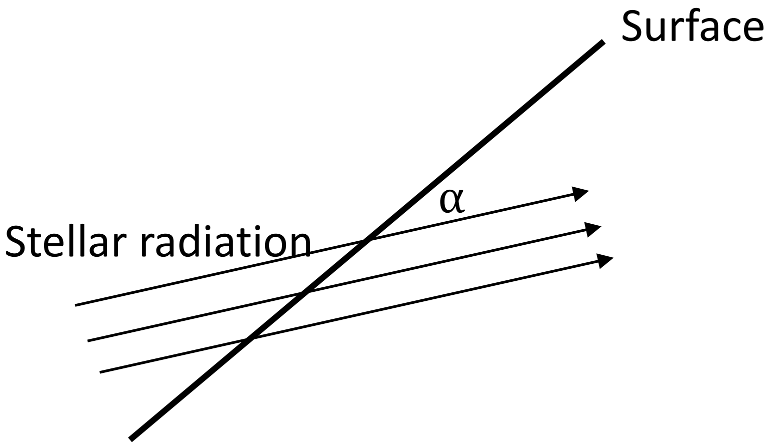
Assuming the disc is optically thick:

$$\frac{1}{2}\alpha \left(\frac{R_*}{R}\right)^2 \sigma T_*^4 = \sigma T_i^4$$

Hence:

$$T_i = \left(\frac{\alpha}{2}\right)^{\frac{1}{4}} \left(\frac{R_*}{R}\right)^{\frac{1}{2}} \mathrm{T}_*$$

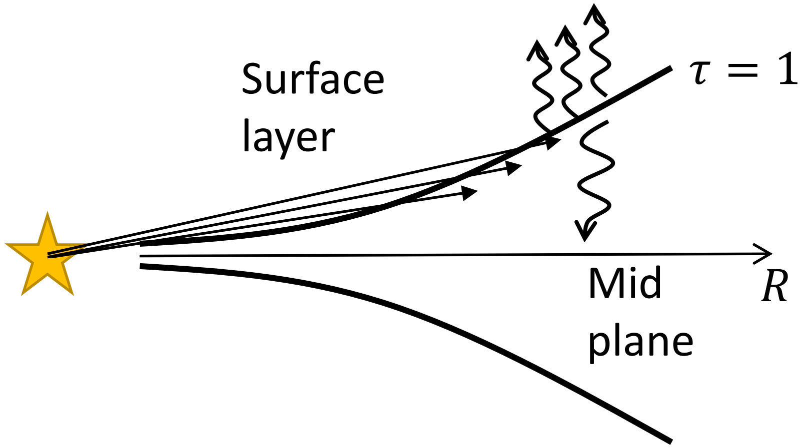


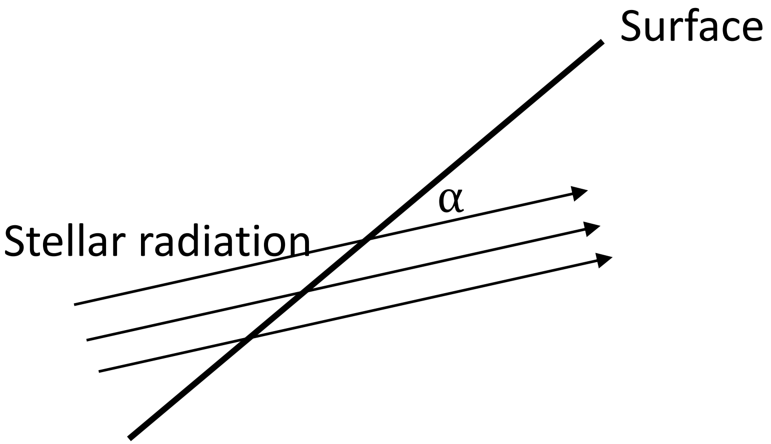


Heating and Cooling

Interior temperature

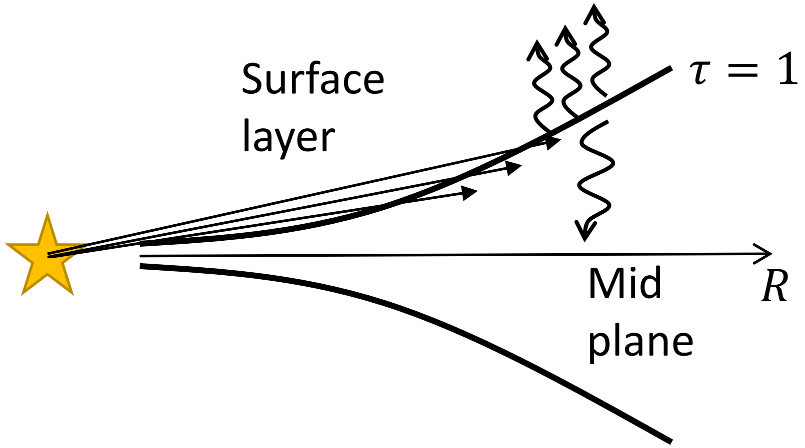
What happens if the disc is not optically thick in the vertical direction?





Heating and Cooling

Interior temperature



What happens if the disc is not optically thick in the vertical direction?

Case 1: Optically thin to the interior's own emission:
Emission from the interior. Solve:

$$\frac{dI_\nu}{dz} = -\kappa_\nu I_\nu + j_{\nu(T_i)} = \kappa_\nu (B_\nu(T_i) - I_\nu)$$

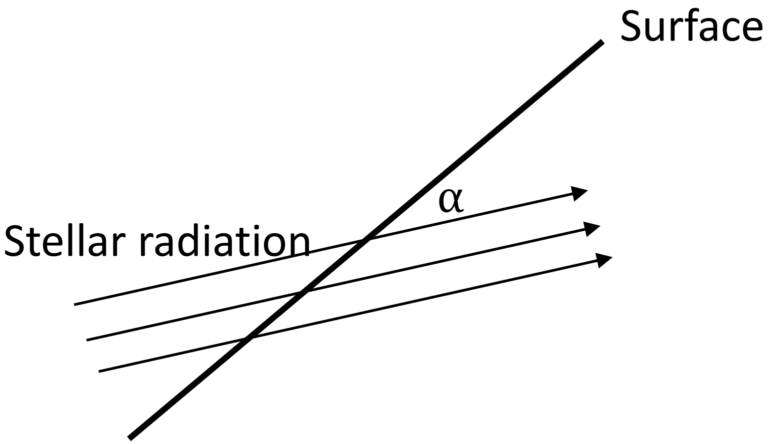
$$I_\nu(\tau) = B_\nu(T_i)[1 - \exp(-\tau)]$$

$$(\tau = \kappa_\nu \Sigma)$$

Integrating over frequency:

$$cooling = \sigma T^4 [1 - \exp(-\tau_i)]$$

where we've assumed $\tau_\nu = \tau_i = \kappa_P\,(T_i)\,\Sigma$



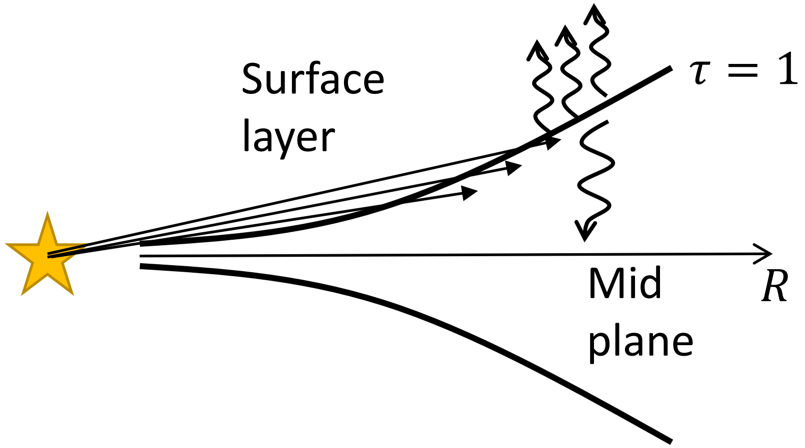
Heating and Cooling

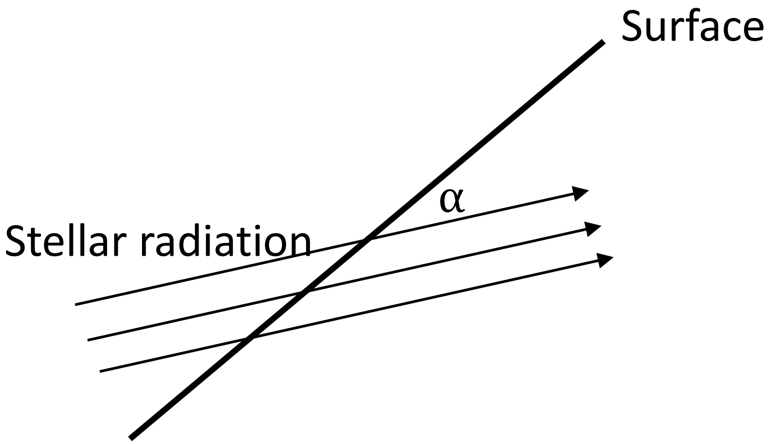
Interior temperature

What happens if the disc is not optically thick in the vertical direction?

Case 1: Optically thin to the interior's own emission:

We only need to add a factor $\psi_i = 1 \ - \exp(\tau_i)$ into the cooling rate.





Heating and Cooling

Interior temperature

What happens if the disc is not optically thick in the vertical direction?

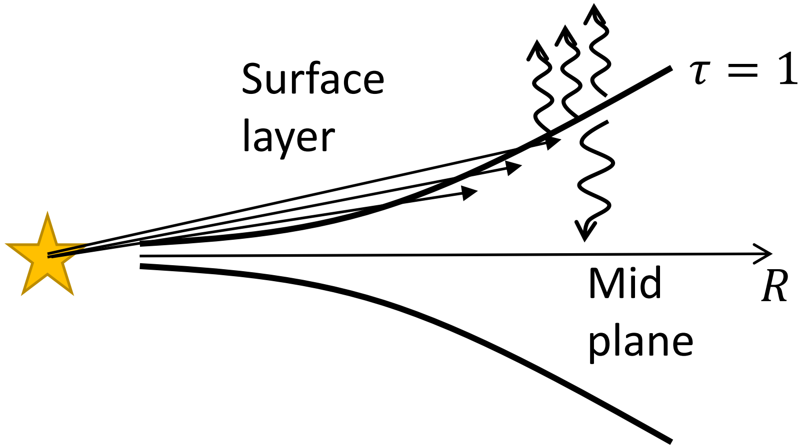
Case 2: Optically thin to the heating radiation

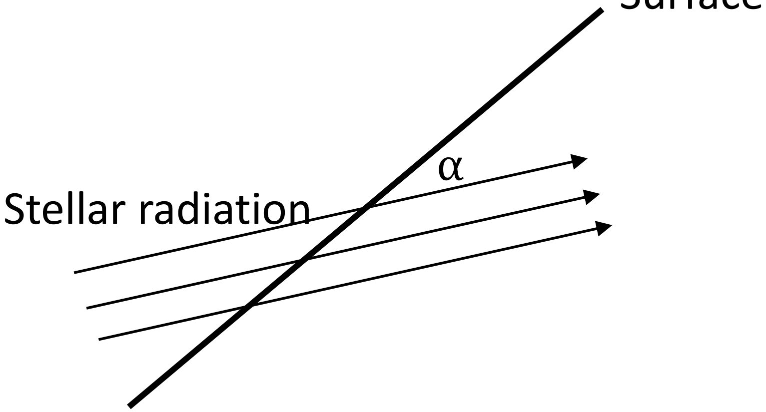
Again we solve:

$$\frac{dI_\nu}{dz} = -\kappa_\nu I_\nu + j_{\nu(T_i)} = \kappa_\nu (B_\nu(T_i) - I_\nu)$$

This time we neglect $B_\nu(T)$:

$$I_\nu(\tau) = I_0 \exp(-\tau)$$





Heating and Cooling

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What happens if the disc is not optically thick in the vertical direction?

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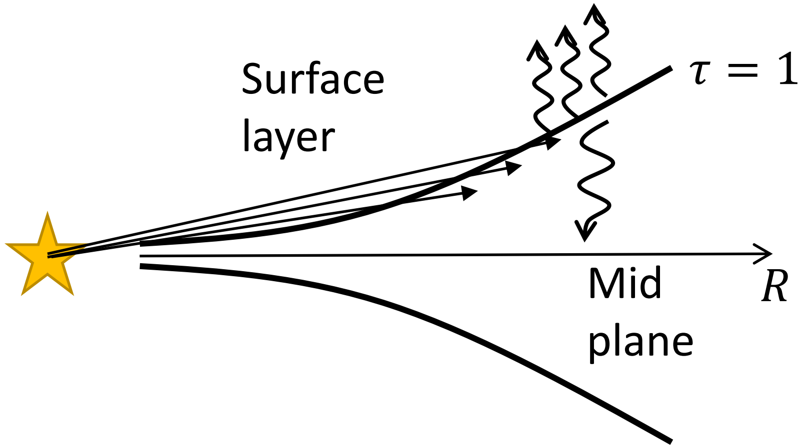
$$\frac{dI_\nu}{dz} = -\kappa_\nu I_\nu + j_{\nu(T_i)} = \kappa_\nu (B_\nu(T_i) - I_\nu)$$

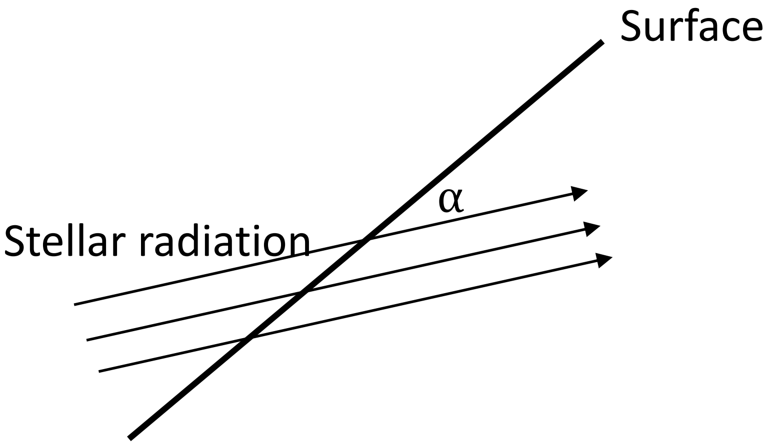
This time we neglect $B_\nu(T)$:

$$I_\nu(\tau) = I_0 \exp(-\tau)$$

This time $\tau = \tau_s = \kappa_P(T_s)$

Energy deposited is $\mathrm{I}_0 - I_\nu(\tau_s)$.





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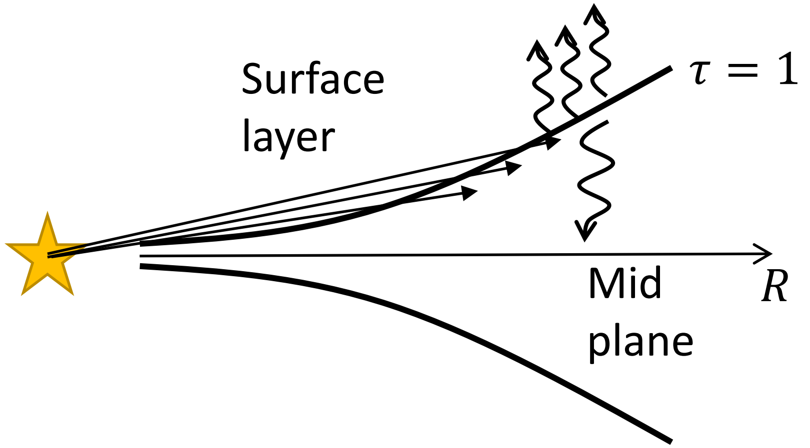
This time $\tau = \tau_s = \kappa_P(T_s)$

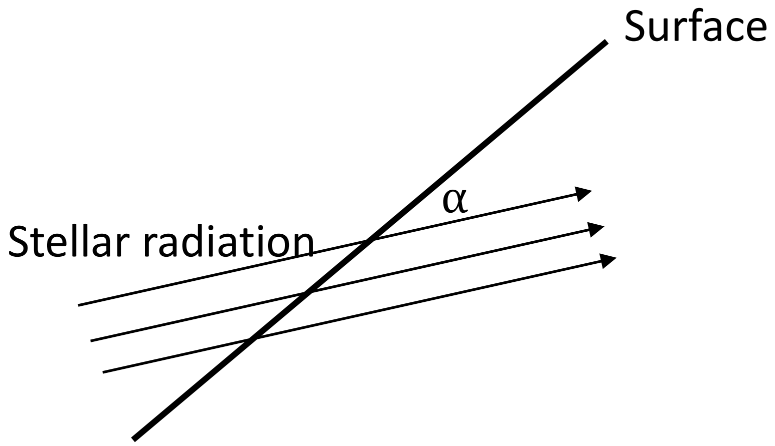
Energy deposited is $\mathrm{I}_0 - I_\nu(\tau_s)$.

Hence we can simply introduce another factor:

$$\psi_s = 1 - \exp(-\tau_s)$$

Into the heating rate





Heating and Cooling

Interior temperature

What happens if the disc is not optically thick in the vertical direction?

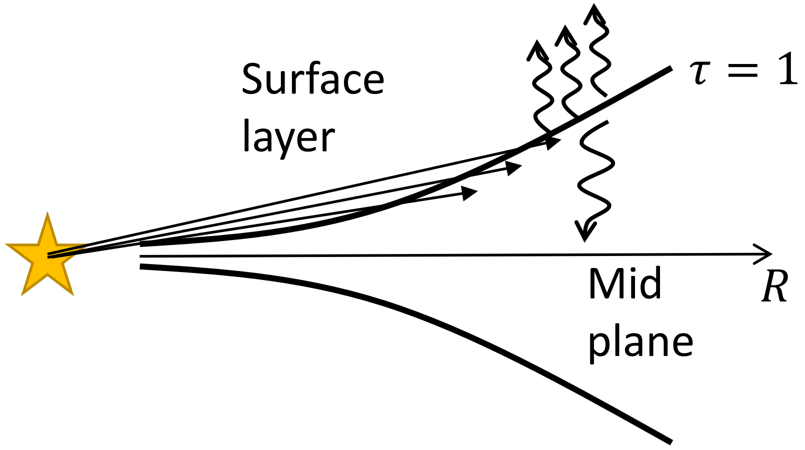
Putting it together:

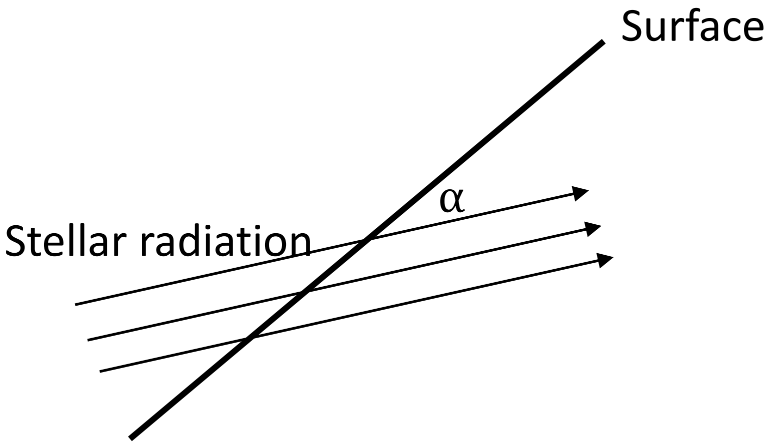
The energy balance for the interior is now:

$$\psi_s \frac{1}{2} \alpha \left(\frac{R_*}{R}\right)^2 \sigma T_*^4 = \psi_i \sigma T_i^4$$

so

$$T_i = \left(\frac{\alpha}{2}\frac{\psi_s}{\psi_i}\right)^{\frac{1}{4}} \left(\frac{R_*}{R}\right)^{\frac{1}{2}} \mathrm{T}_*$$





Heating and Cooling

Interior temperature

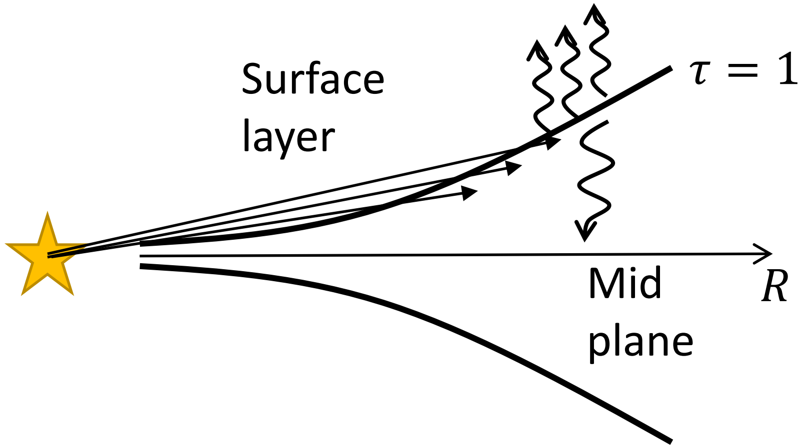
Can we construct a model for $T(z)$ from these two layers?

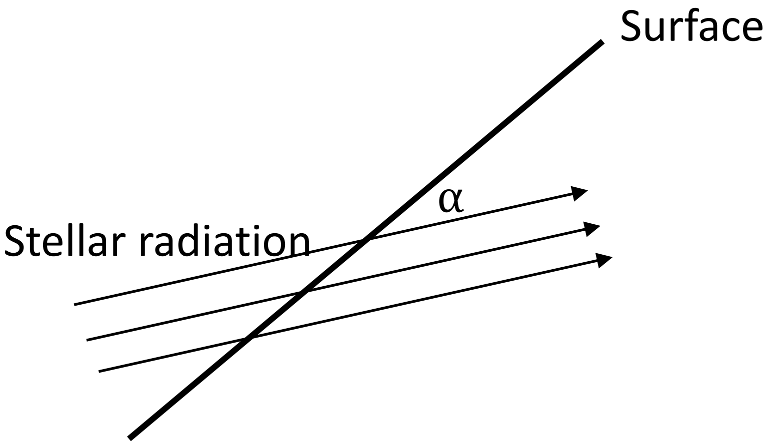
Surface:

$$T_s = \left(\frac{\kappa_P(T_*)}{\kappa_P(T_s)}\right)^{\frac{1}{4}} \left(\frac{R_*}{2R}\right)^{\frac{1}{2}} T_*$$

Interior:

$$T_i = \left(\frac{\alpha}{2}\frac{\psi_s}{\psi_i}\right)^{\frac{1}{4}} \left(\frac{R_*}{R}\right)^{\frac{1}{2}} \mathrm{T}_*$$





Heating and Cooling

Interior temperature

Can we construct a model for $T(z)$ from these two layers?

Modifying the surface expression:

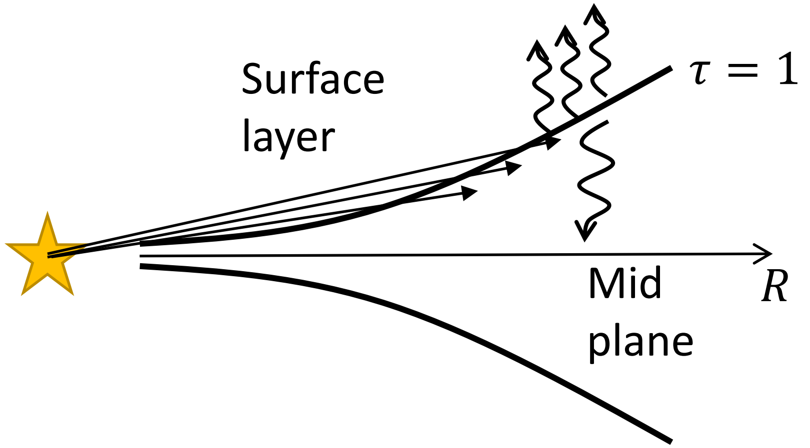
As we move along a ray closer to the interior the stellar flux drops due to absorption.

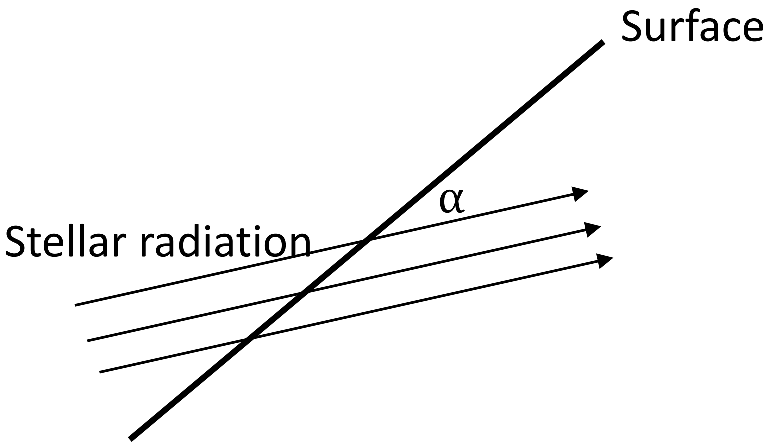
Hence:

$$\left(\frac{R_*}{R}\right)^2 \kappa_P(T_*)\,\sigma T_*^4 \exp(-\tau_*/\alpha) = 4\kappa_P(T_s)\sigma T_s^4$$

Or

$$\boxed{T_s = \left(\frac{\kappa_P(T_*)}{\kappa_P(T_s)}\right)^{\frac{1}{4}} \left(\frac{R_*}{2R}\right)^{\frac{1}{2}} \exp\left(-\frac{\tau_*}{4\alpha}\right) T_*}$$





Heating and Cooling

Interior temperature

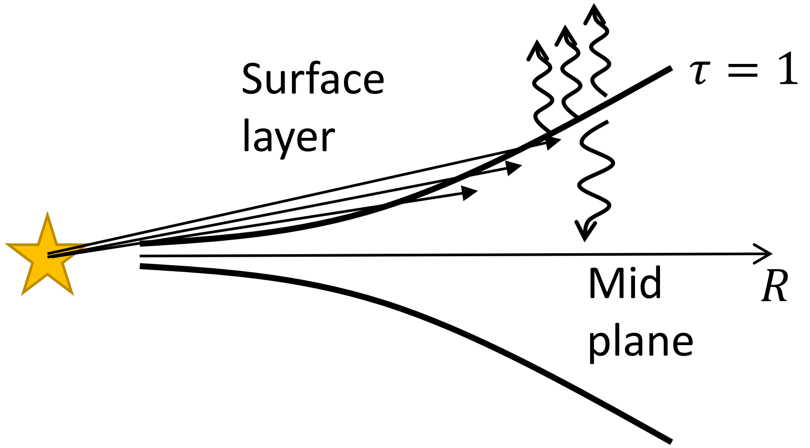
Can we construct a model for $T(z)$ from these two layers?

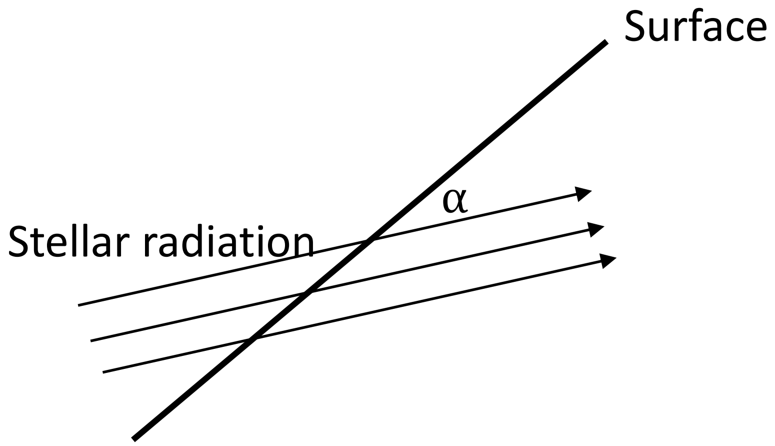
A combined expression:

Need to add the surface and internal together. Hence:

$$T(z)^4 = T_s(z)^4 + T_i^4$$

$$\boxed{T(z)^4 = \left[\frac{1}{4}\frac{\kappa_P(T_*)}{\kappa_P(T_s)}\exp(-\tau_*/\alpha) + \frac{\alpha}{2}\frac{\psi_s}{\psi_i}\right]\left(\frac{\mathrm{R}_*}{\mathrm{R}}\right)^2 \mathrm{T}_*^4}$$





Heating and Cooling
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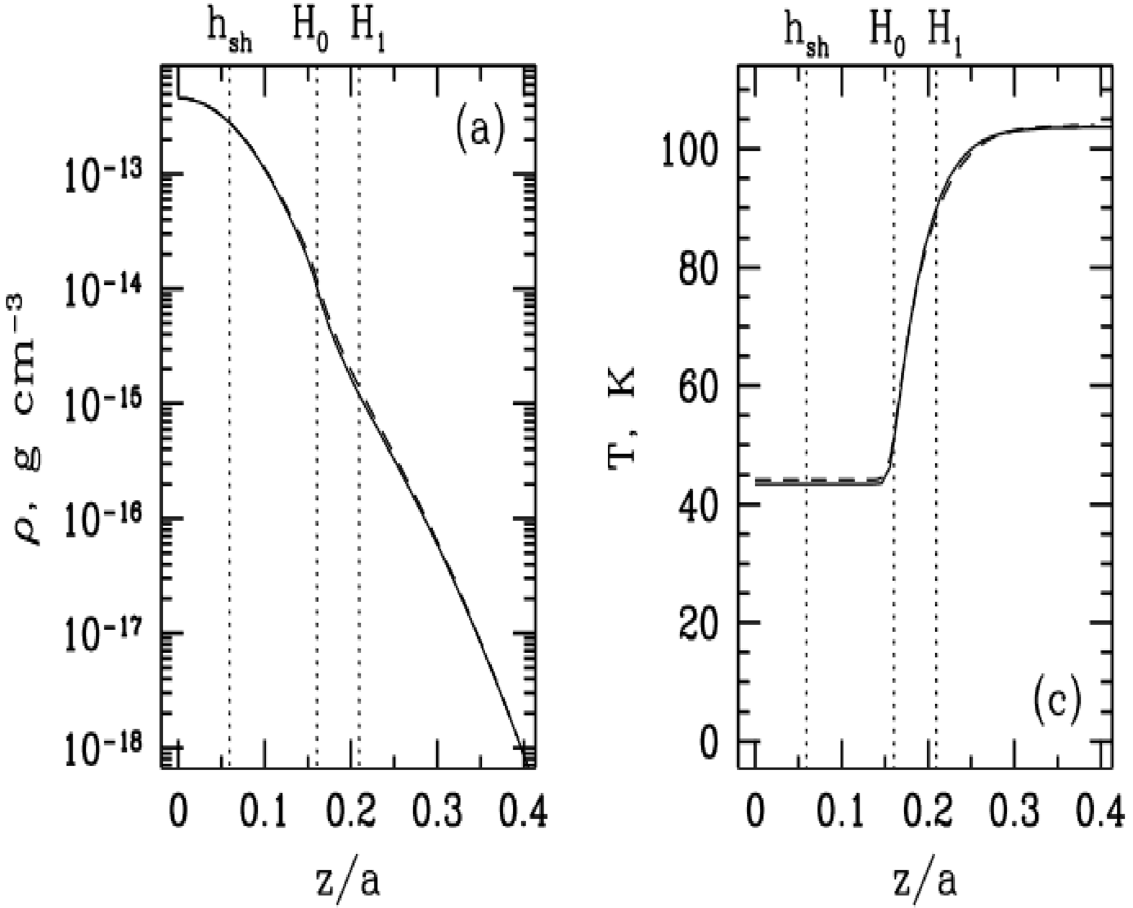
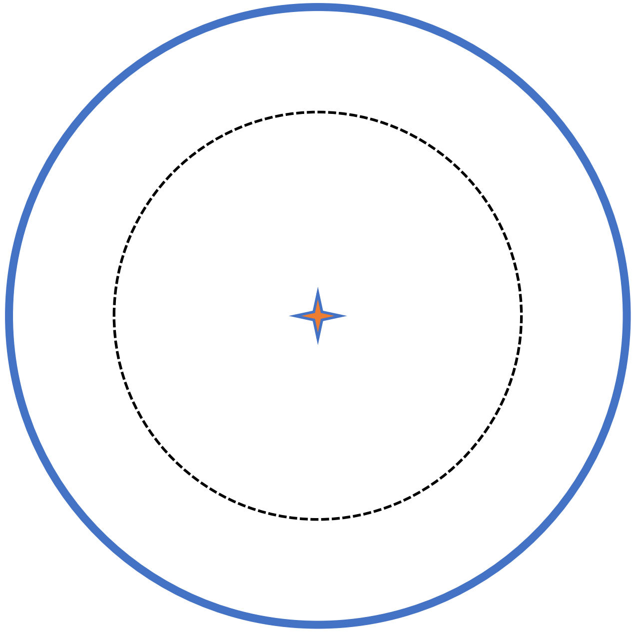
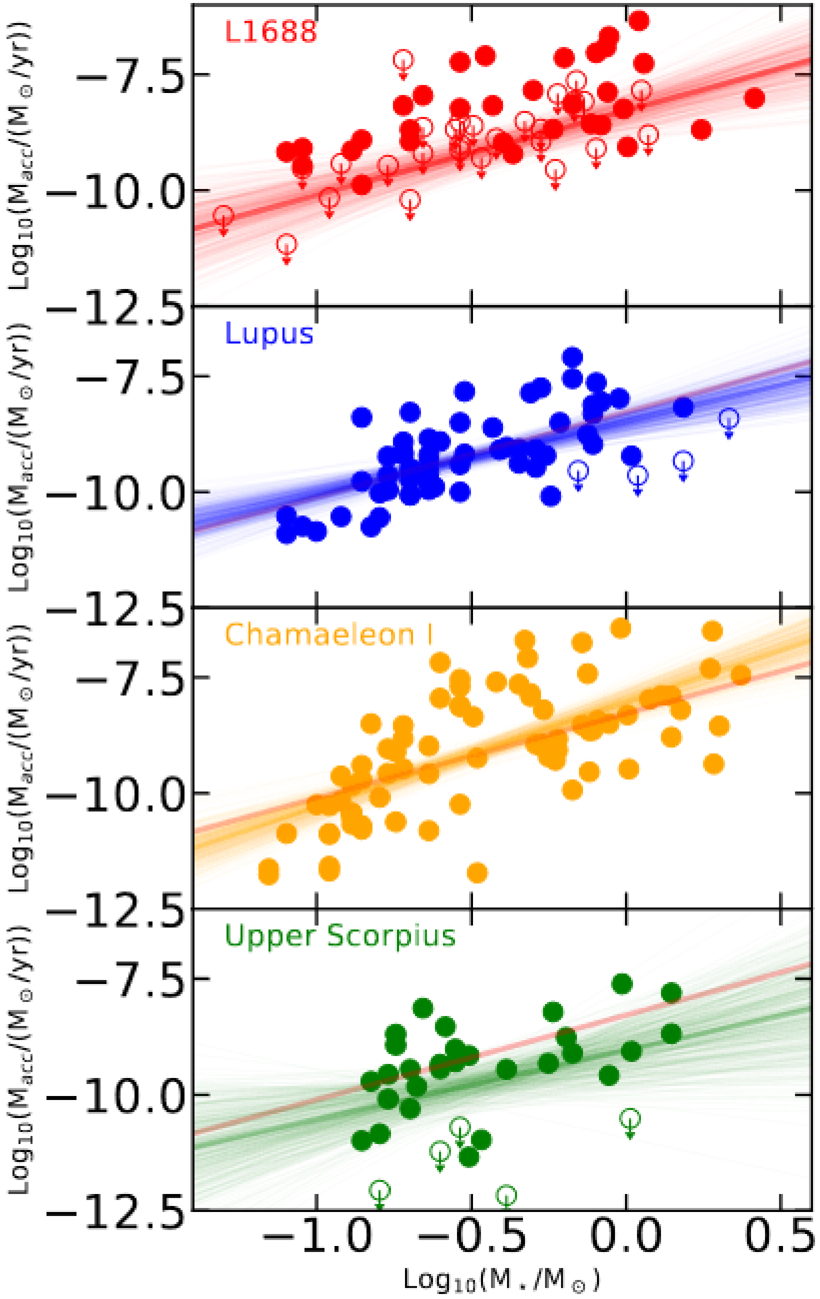


Rafikov+ (2006)

Heating and Cooling

Viscous Heating





Heating and Cooling
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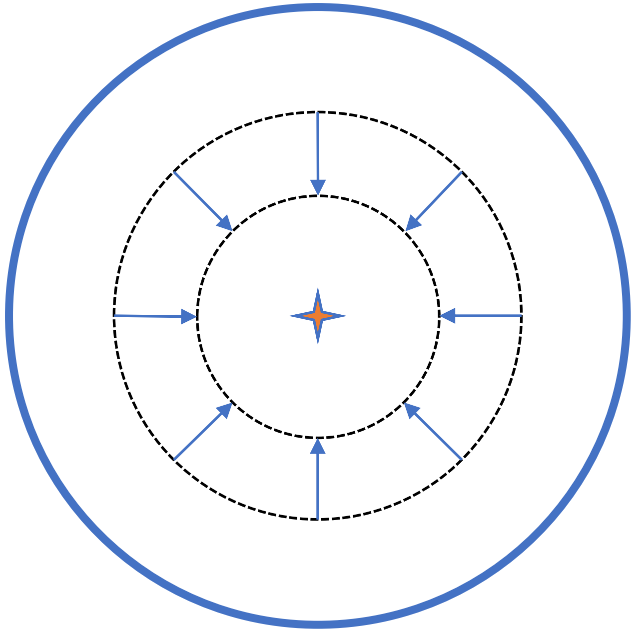
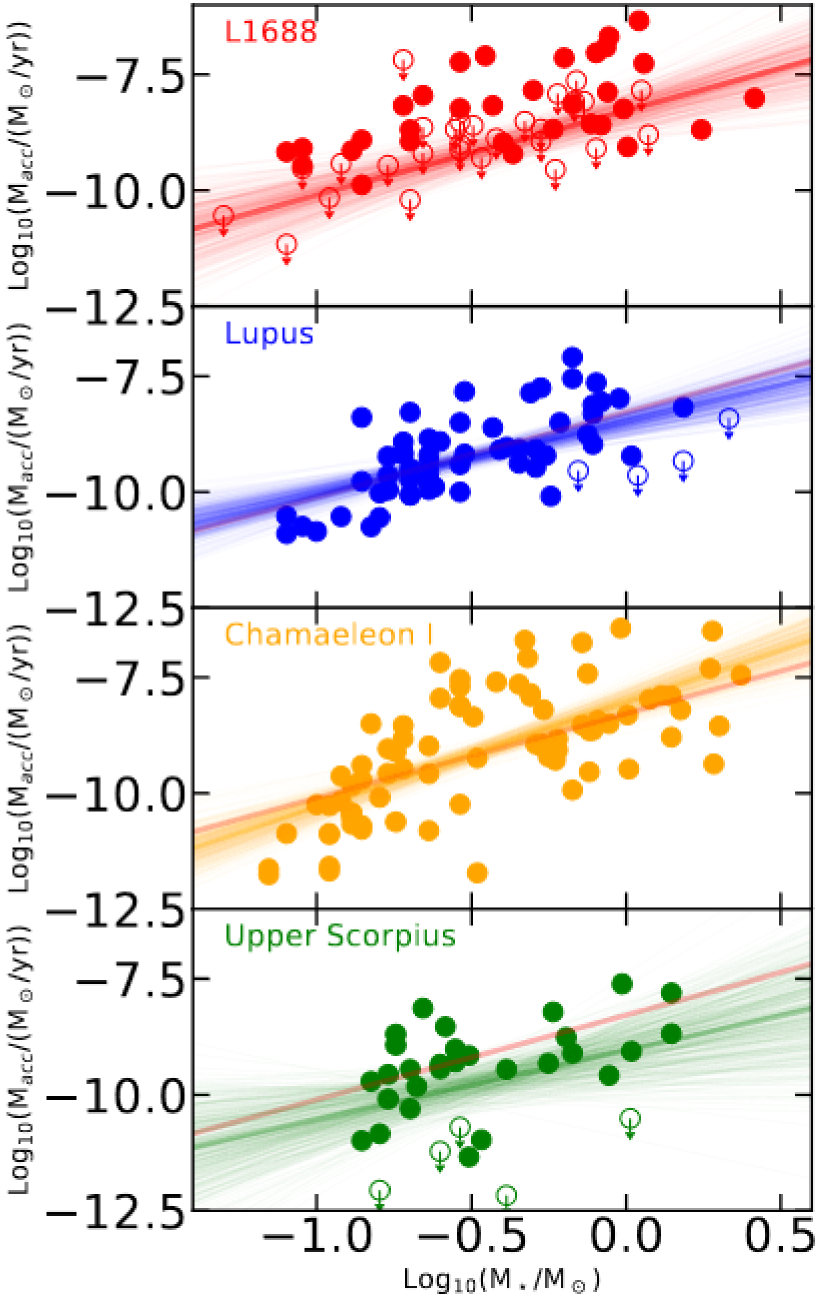


Accretion requires energy loss:
➔ Heating

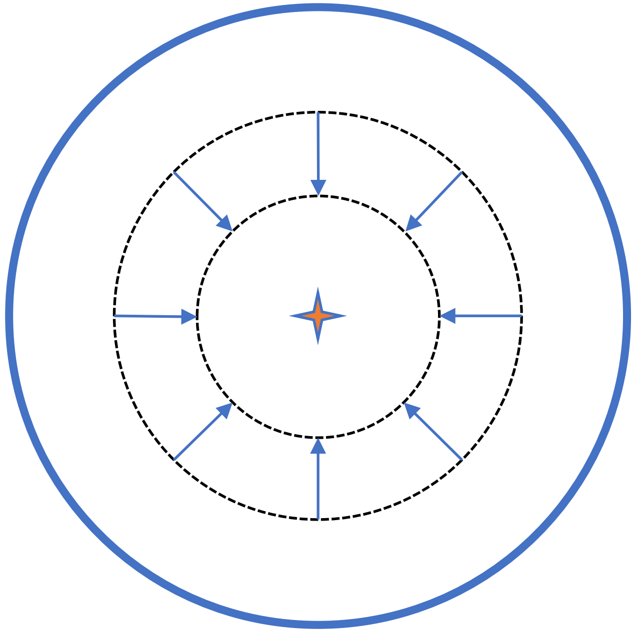


Heating and Cooling

Viscous Heating

Assume accretion is driven by viscosity:

$$Q^{+} = \rho\nu \left| \frac{d\Omega}{dr} \right|^{2}$$



Heating and Cooling

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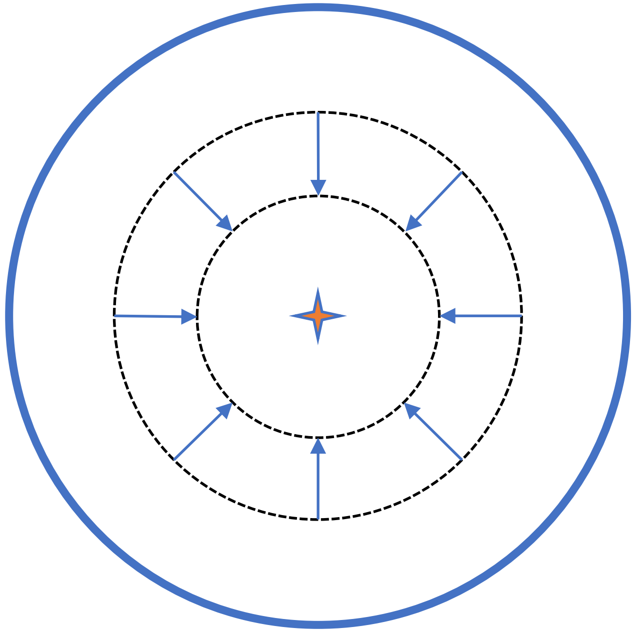
Assume accretion is driven by viscosity:

$$Q^{+} = \rho\nu\left|\frac{d\Omega}{dr}\right|^{2}$$

Integrate vertically and assume Keplerian shear:

$$Q^{+} = \frac{9}{4}\nu\Sigma\Omega^{2} = \frac{3}{4\pi}\dot{M}\Omega^{2}$$



Heating and Cooling
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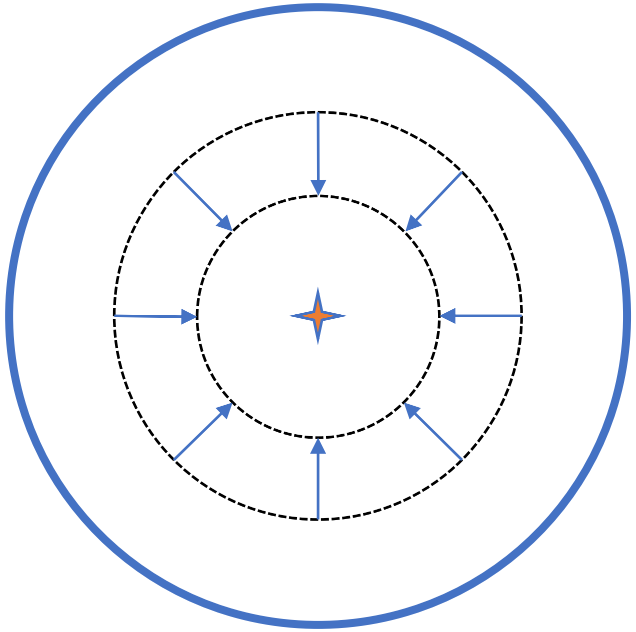
Balance heating/cooling to compute effective temperature:

$$\frac{3}{4\pi}\dot{M}\Omega^2 = 2\sigma T_{eff}^4$$

(cooling from both sides of the disc)



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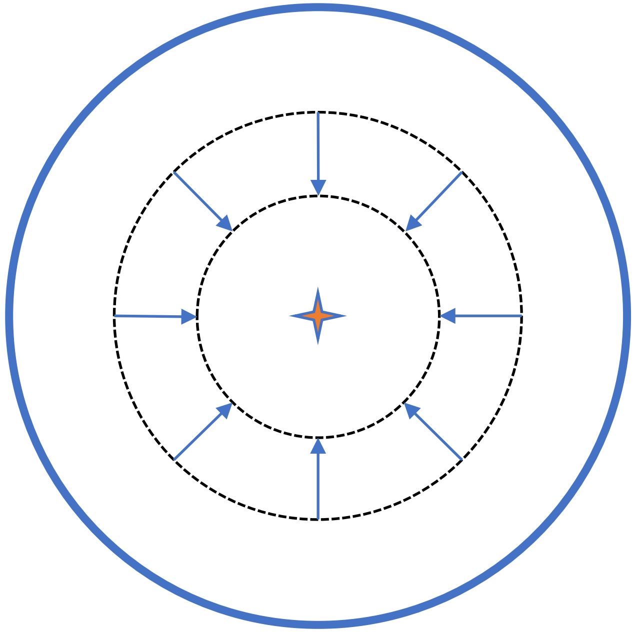
$$T_{eff} \sim r^{-3/4}, \qquad T_{irr} \sim r^{-1/2}$$

Viscous heating is most important close to the star



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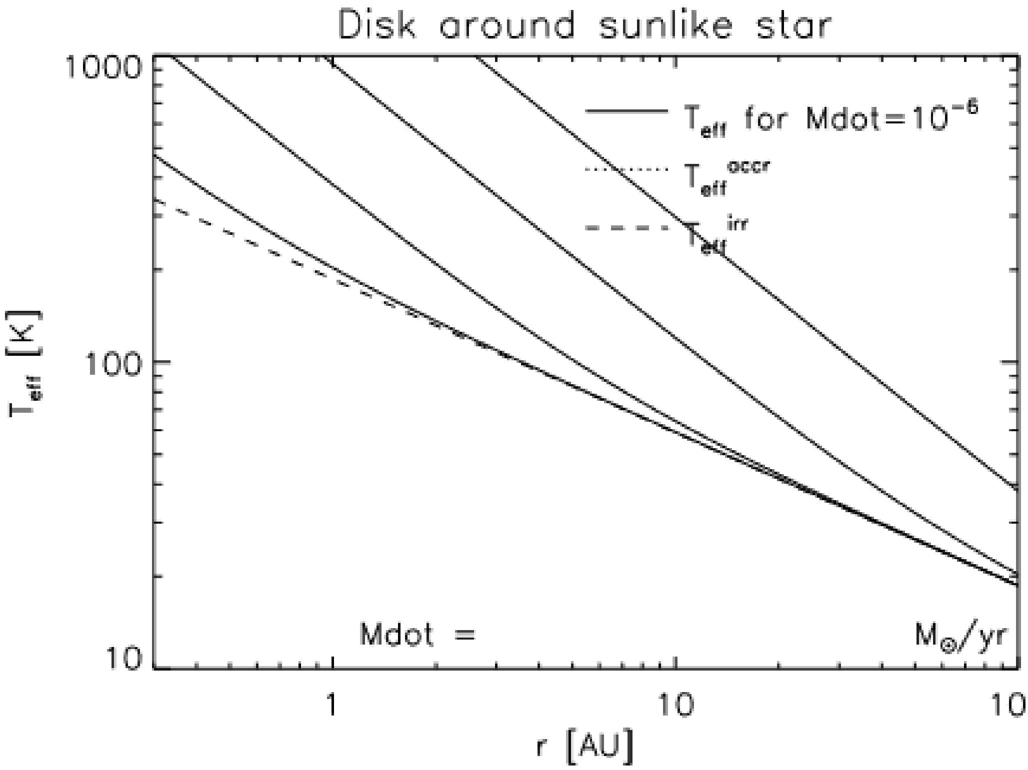


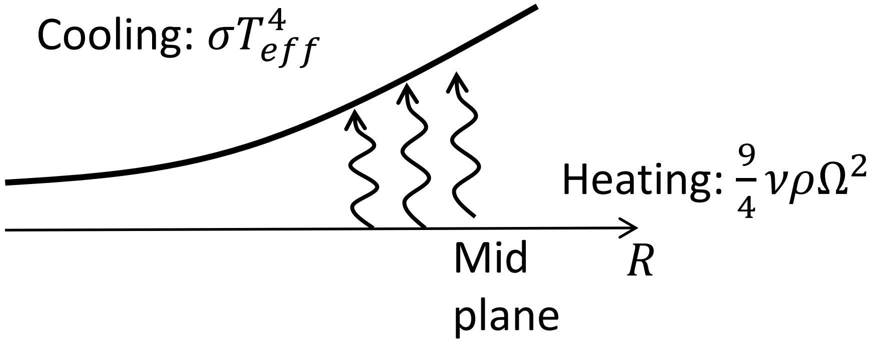
Courtesy of Kees. Dullemond

Heating and Cooling

Viscous Heating

Vertical temperature structure

- Most heating occurs near the mid-plane
- The cooling occurs at the disc surface
- Disc is optically thick close to the star



Heating and Cooling

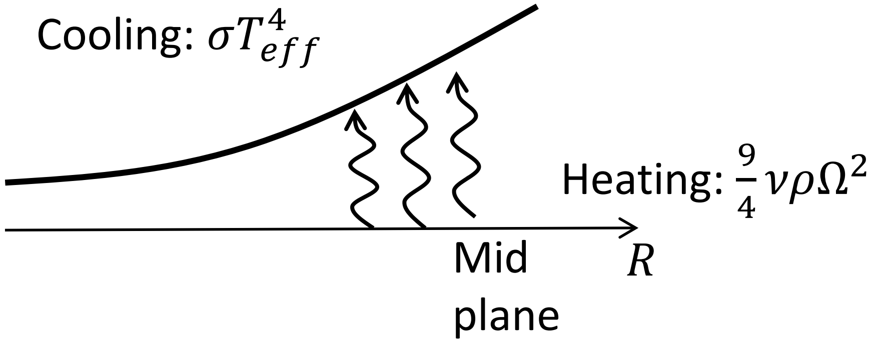
Viscous Heating

Vertical temperature structure

Consider diffusion of radiation from mid-plane to surface

$$Flux = -\frac{4\pi}{3\rho\kappa}\frac{dJ}{dz}$$

Optically thick so $J = \sigma T^4/\pi$.



Heating and Cooling

Viscous Heating

Vertical temperature structure

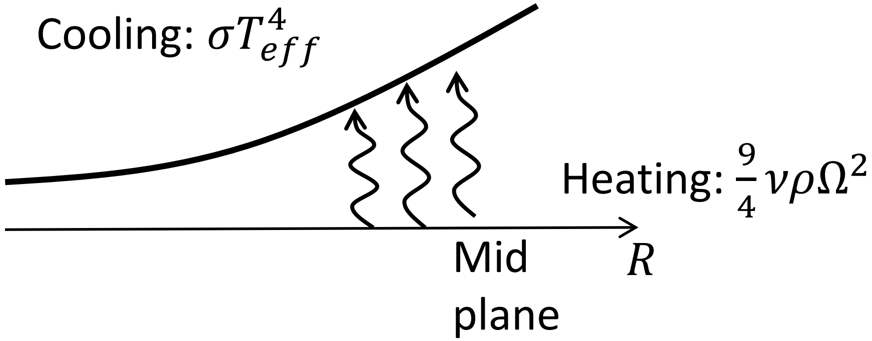
Consider diffusion of radiation from mid-plane to surface

$$Flux = -\frac{4\pi}{3\rho\kappa}\frac{dJ}{dz}$$

Optically thick so $J = \sigma T^4/\pi$.

Approximate all of the heating being at the midplane, so the flux is just Q^+:

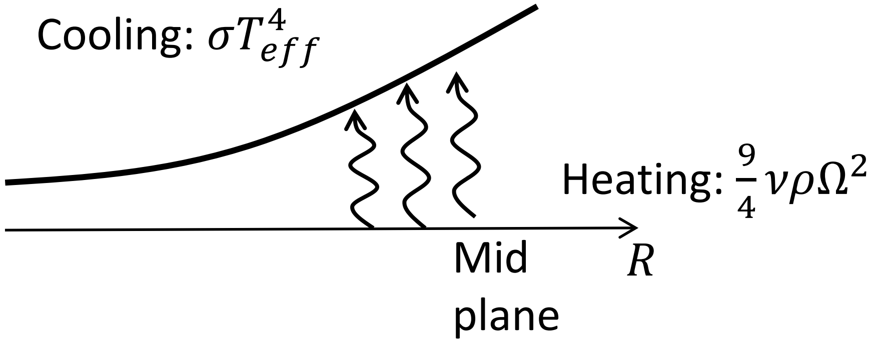
$$Q^+ = -4\frac{\sigma}{3}\frac{dT^4}{d\tau}$$



Heating and Cooling

Viscous Heating

Vertical temperature structure



Previous expression:

$$Q^+ = -4\frac{\sigma}{3}\frac{dT^4}{d\tau}$$

Solve:

$$T_{eff}^4 \ - T(z)^4 = -\frac{3Q^+}{4\sigma}\tau(z)$$

$\tau(z)$ is measured from the surface

Heating and Cooling

Viscous Heating

Vertical temperature structure

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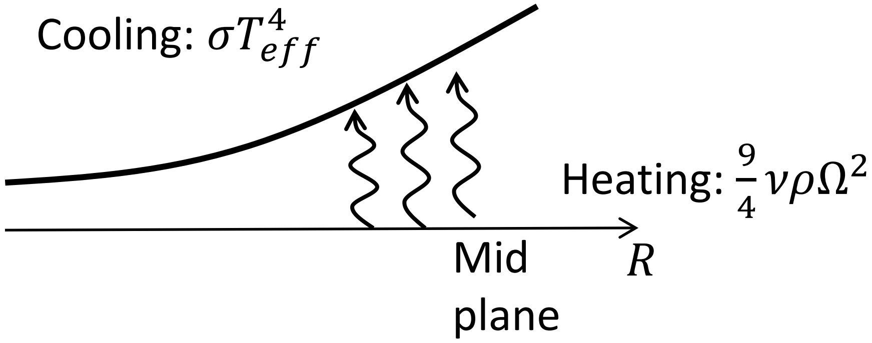
Solve:

$$T_{eff}^4 - T(z)^4 = -\frac{3Q^+}{4\sigma}\tau(z)$$

But $Q^+ = 2\sigma T_{eff}^4$

$$T(z)^4 = \left[\frac{1}{2} + \frac{3\tau(z)}{4}\right]\frac{Q^+}{\sigma}$$

$\tau(z)$ is measured from the surface



Heating and Cooling

Viscous Heating

Vertical temperature structure

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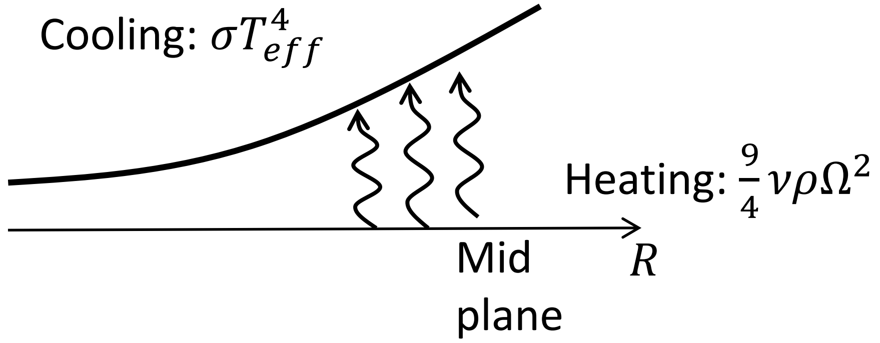
But $\mathrm{Q}^+ = 2\sigma T_{eff}^4$

$$T(z)^4 = \left[\frac{1}{2} + \frac{3\tau(z)}{4}\right]\frac{Q^+}{\sigma}$$

Hence the mid-plane temperature is:

$$T_m^4 = \left[\frac{1}{2} + \frac{3}{8}\kappa\Sigma\right] \cdot \frac{3}{4\pi}\frac{\dot{M}\Omega^2}{\sigma}$$



$\tau(z)$ is measured from the surface

Heating and Cooling

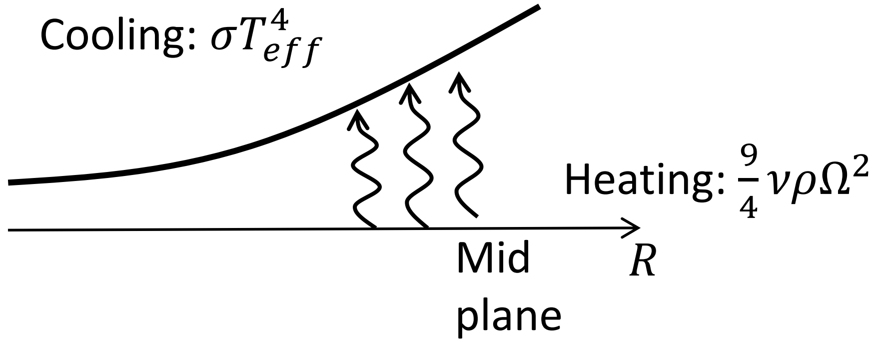
Viscous Heating

Vertical temperature structure

Mid-plane temperature

$$T_m^4 = \left[\frac{1}{2} + \frac{3}{8}\kappa\Sigma\right] \cdot \frac{3}{4\pi}\frac{\dot{M}\Omega^2}{\sigma}$$

Here we've assumed the disc is optically thick.



Heating and Cooling

Viscous Heating

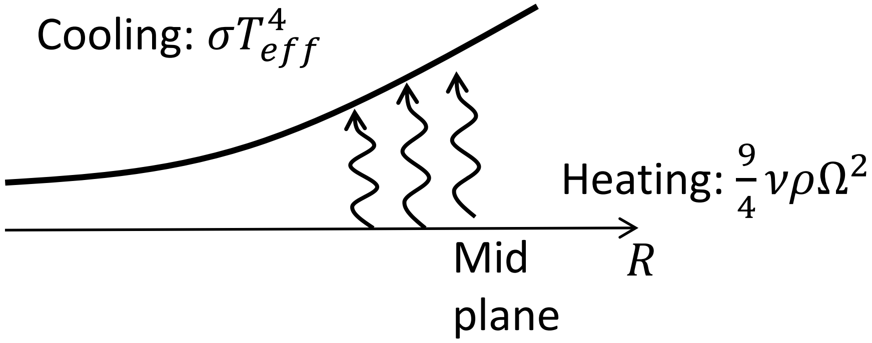
Vertical temperature structure

Mid-plane temperature

$$T_m^4 = \left[\frac{1}{2} + \frac{3}{8}\kappa\Sigma\right] \cdot \frac{3}{4\pi}\frac{\dot{M}\Omega^2}{\sigma}$$

The optically thin limit is easy to obtain:

$$Q^+ = \kappa\Sigma\, \sigma T^4$$



Heating and Cooling

Viscous Heating

Vertical temperature structure

Mid-plane temperature

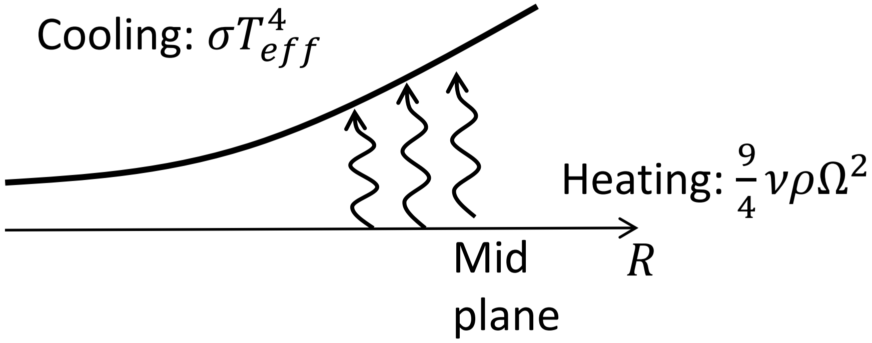
$$T_m^4 = \left[\frac{1}{2} + \frac{3}{8}\kappa\Sigma\right] \cdot \frac{3}{4\pi}\frac{\dot{M}\Omega^2}{\sigma}$$

The optically thin limit is easy to obtain:

$$Q^+ = \kappa\Sigma\, \sigma T^4$$

We can interpolate between the two limits:

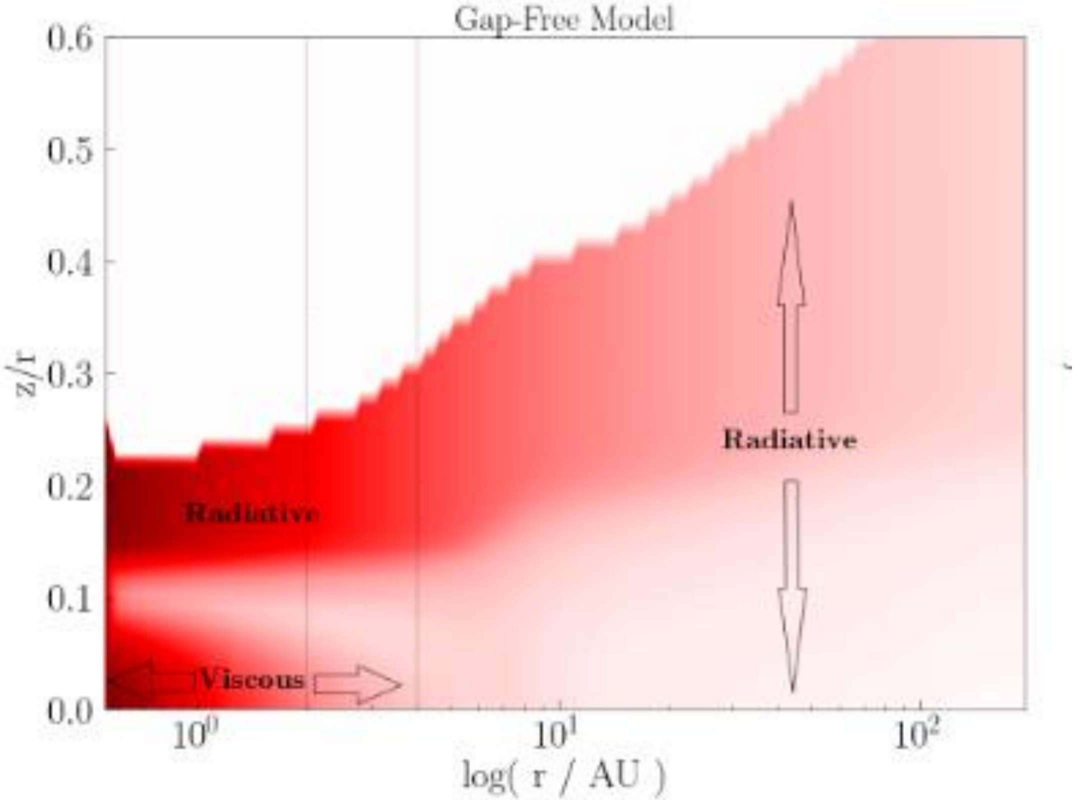
$$T_m^4 = \left[\kappa\Sigma + \ 2 + \frac{8}{3\kappa\Sigma}\right]^{-1} \cdot \frac{3}{4\pi}\frac{\dot{M}\Omega^2}{\sigma}$$



Combine viscous heating and irradiation by adding T^4 together

Summary

- We've described the structure including:
  - Stellar irradiation
  - Viscous heating
  - Cooling

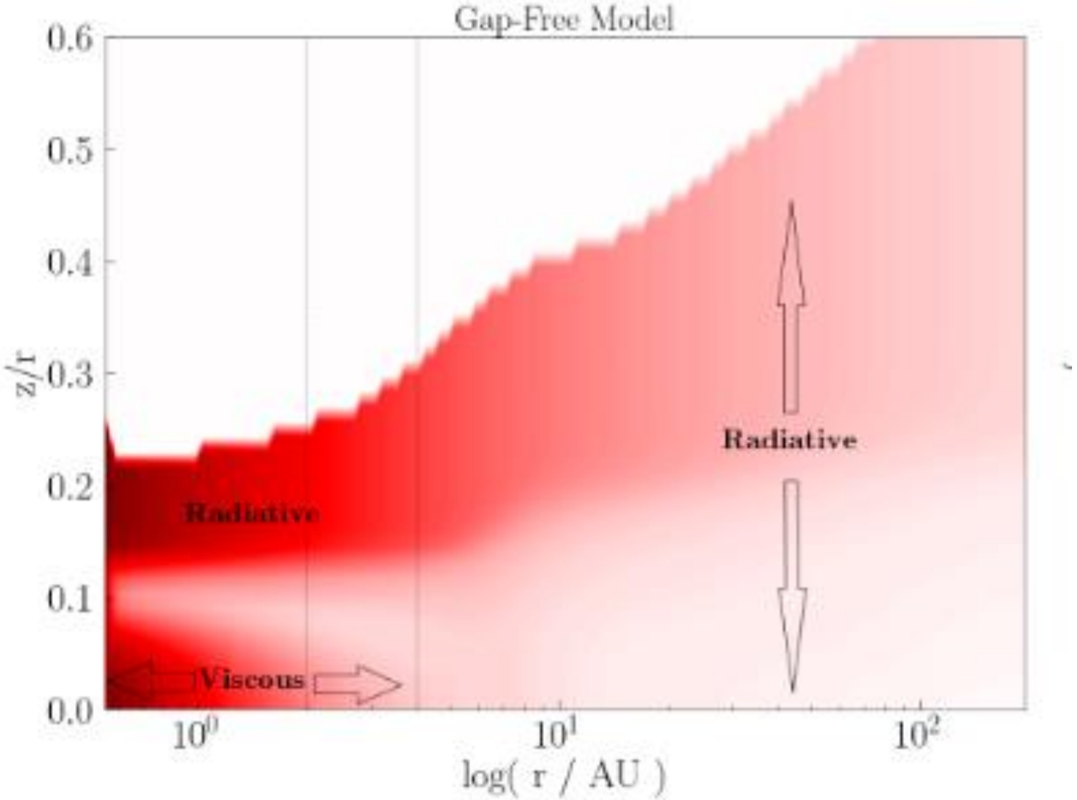


MCRT Model (Broome+ 22)

Summary

- We've described the structure including:
  - Stellar irradiation
  - Viscous heating
  - Cooling

- Several limitations:
  - We Neglected scattering
  - Don't know how to treat shadows



MCRT Model (Broome+ 22)

References

- Chiang & Goldreich 1997
- Dullemond, Dominik & Natta 2001
- Rafikov & De Colle 2006
- Guillot (2010) – A similar model but for planetary atmospheres

For a more detailed model see:

- D’Alessio et al. 1998

How do we avoid these issues in practice?

RADMC-3D Version 2.0

RADMC-3D is a code package for diagnostic radiative transfer calculations in astronomy and astrophysics. It calculates, for a given geometrical distribution of gas and/or dust, what its images and/or spectra look like when viewed from a certain angle, allowing modelers to compare their models with observed data.

Typical applications are protoplanetary disks, circumstellar envelopes, dusty molecular clouds, dusty tori around AGN and models of galaxies. But the code is flexible and can also be applied to other kinds of objects.

The code package is well documented and has numerous simple examples that can be used as templates for one's own models.

The RADMC-3D code is freely available and open source. It runs on linux and OS X. The main code is written in Fortran 90, but all interaction with the code is done through Python interfaces.