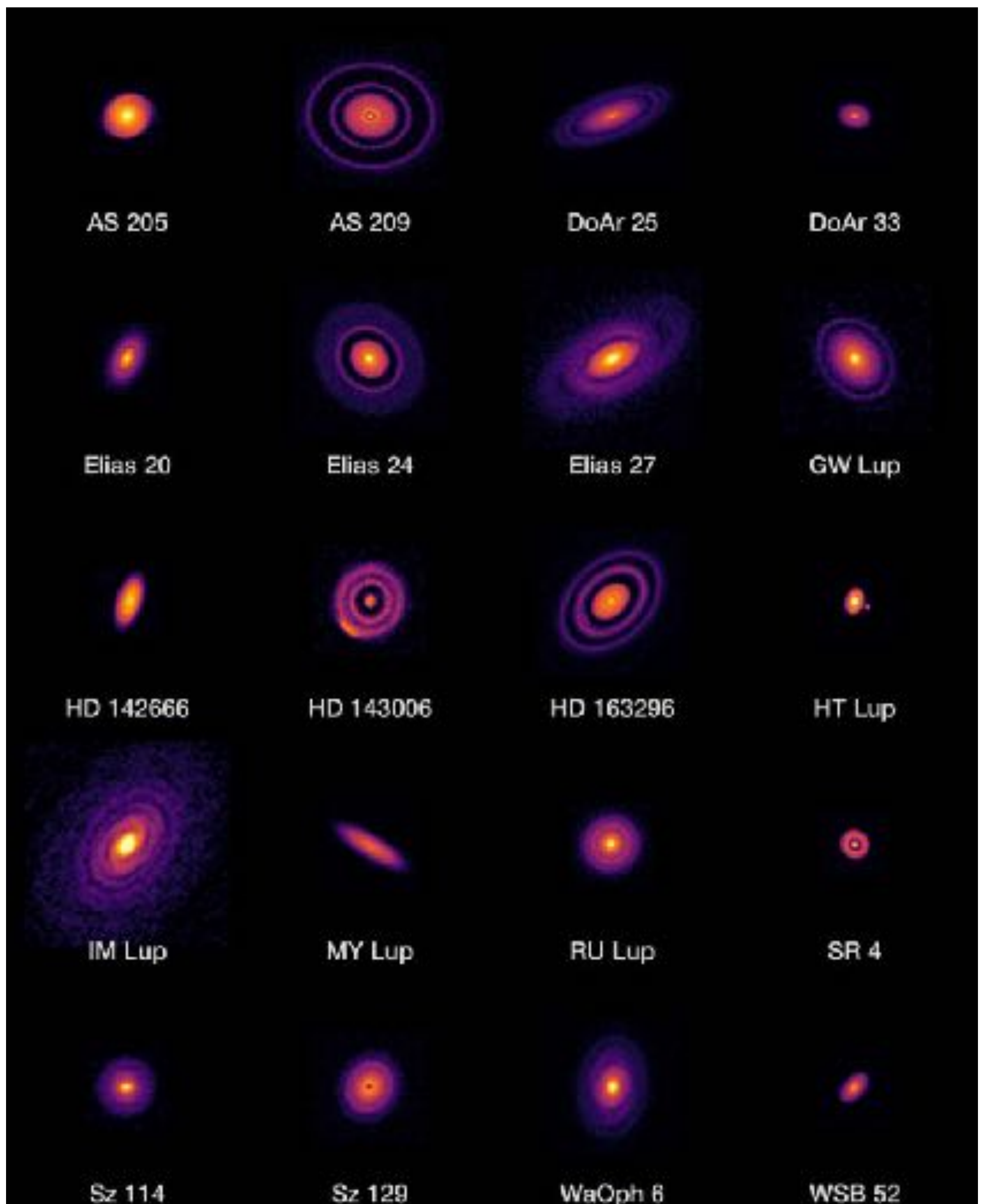
Observing planet forming disks with ALMA

Basics of interferometry II and disk kinematics

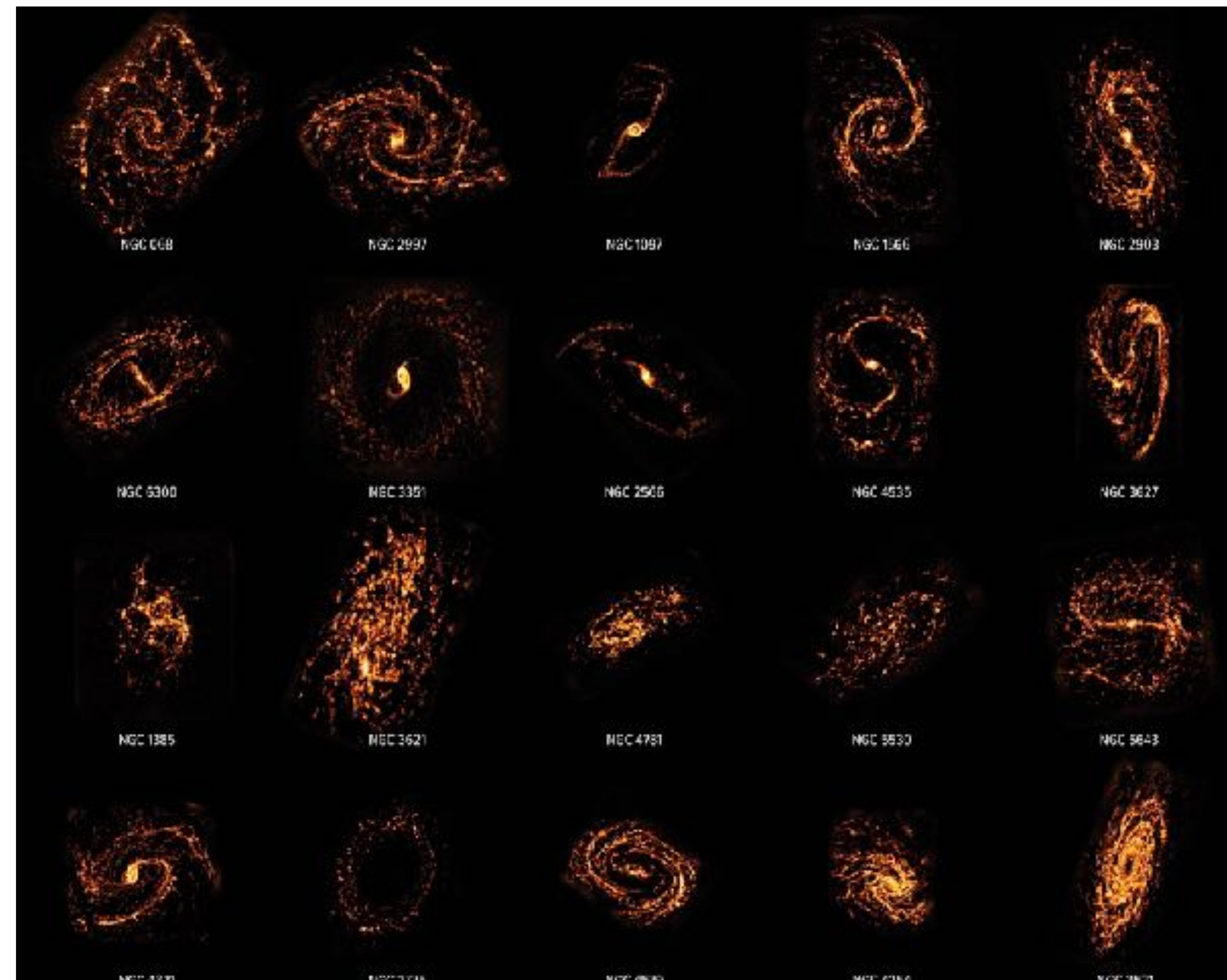
Stefano Facchini

Image-reconstruction

How to obtain an image after measuring V(u,v)?



Andrews+2018

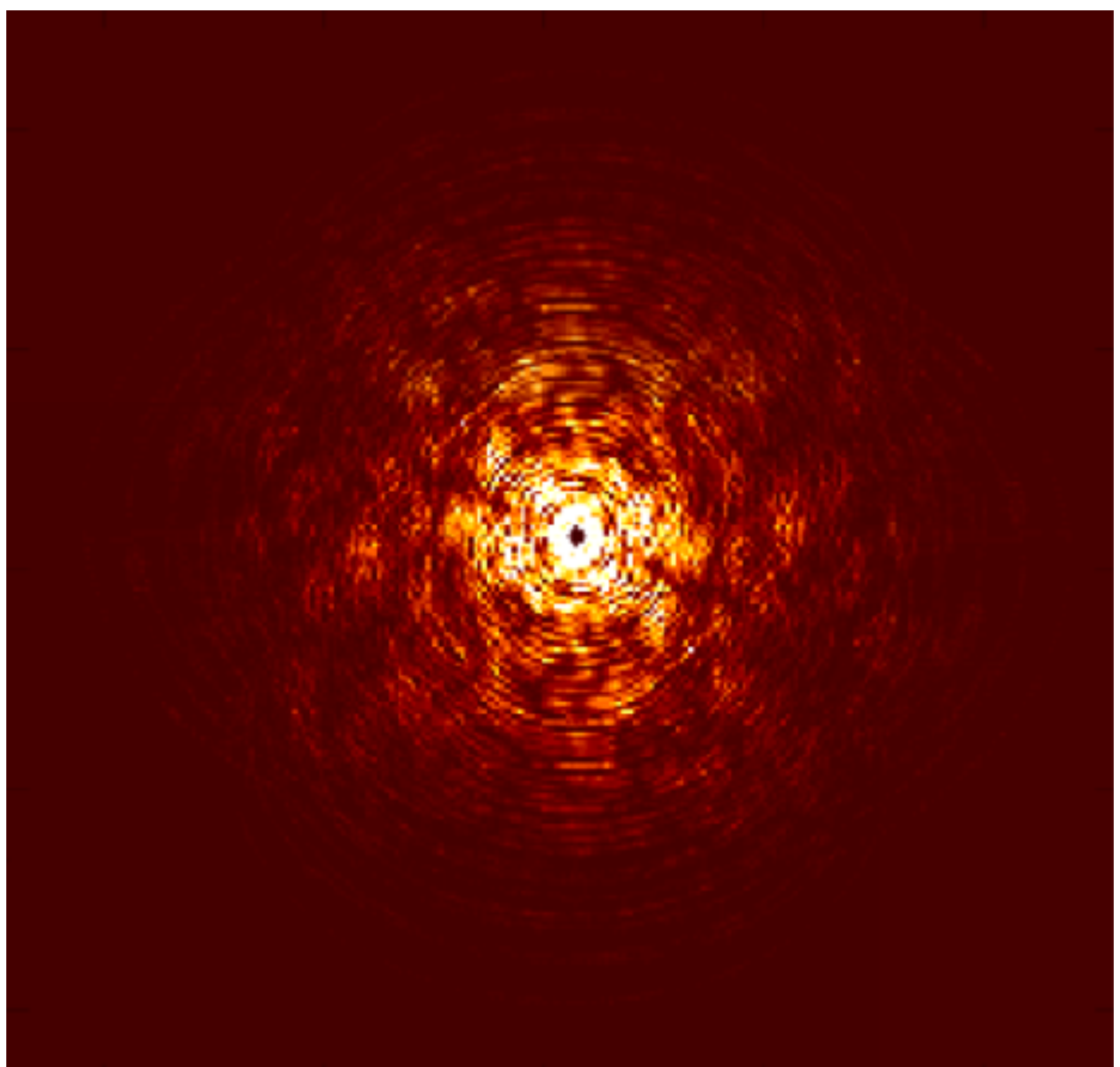


Dagnello+2021

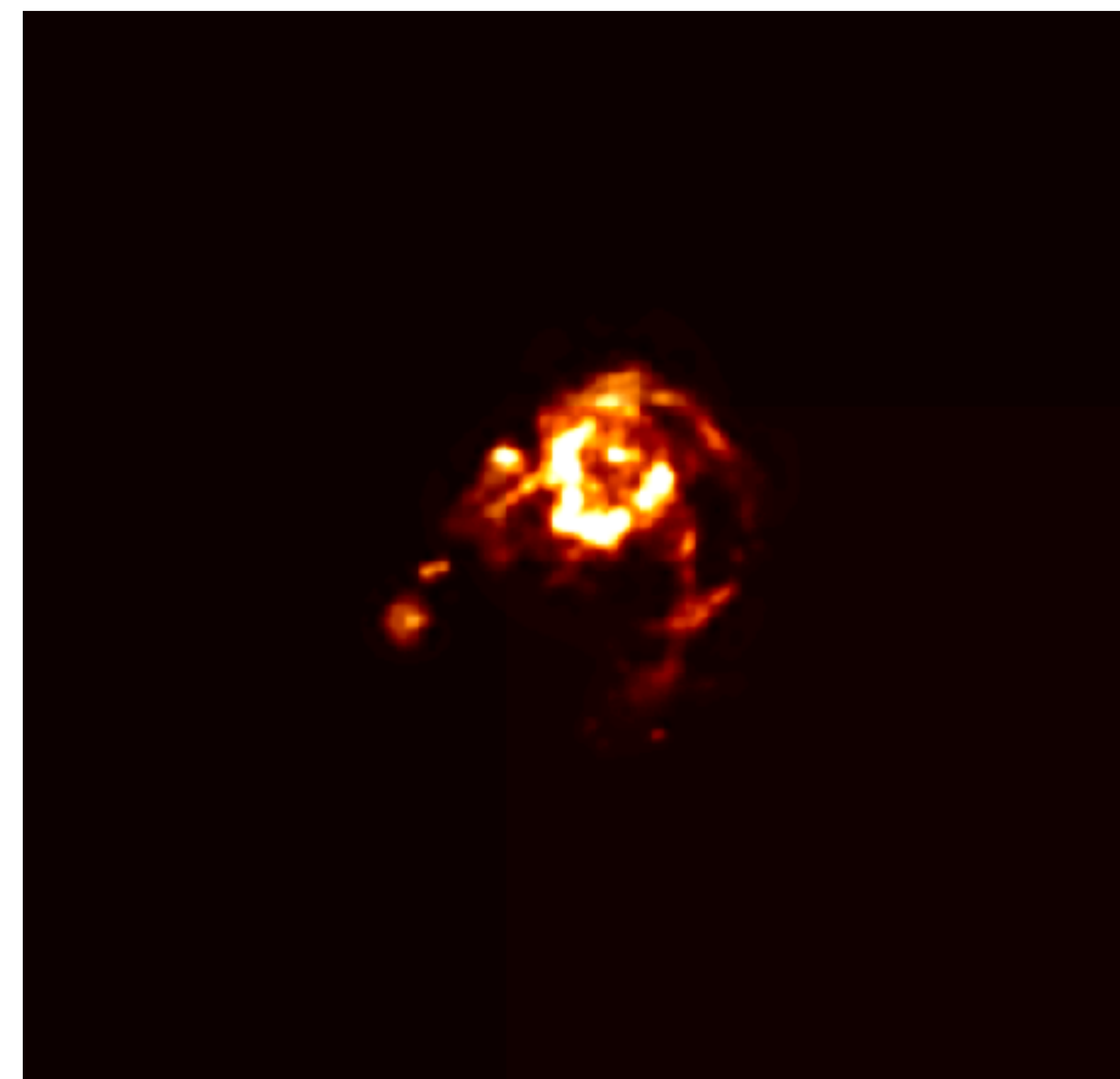
How to we go from V(u,v) to I(x,y)?

$$V(u,v) = \int I(x,y) e^{-2\pi i(ux+vy)} dxdy$$

V(u,v)



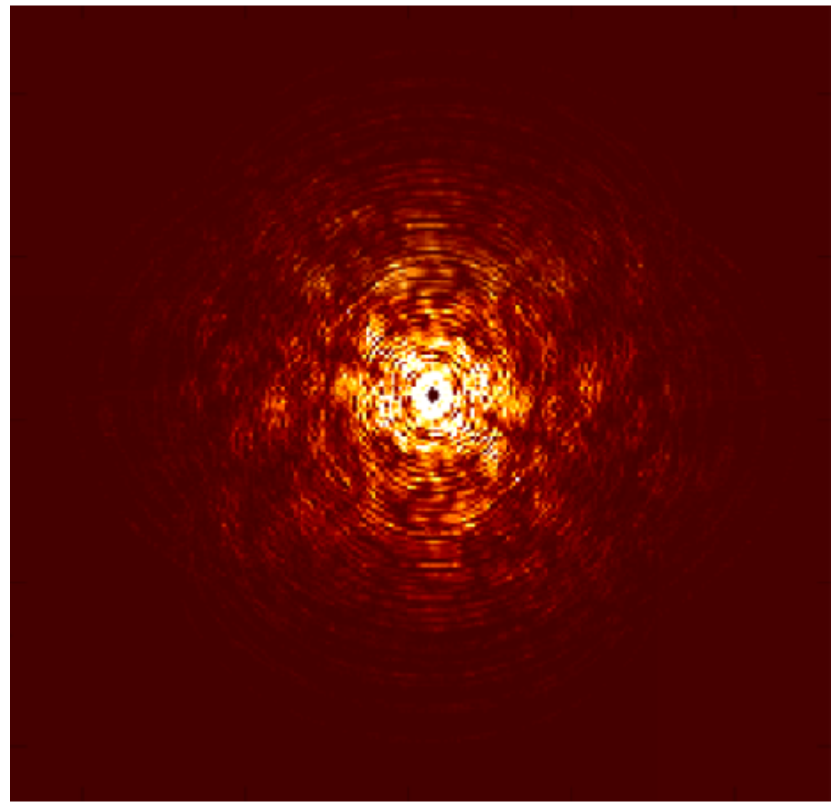
I(x,y)

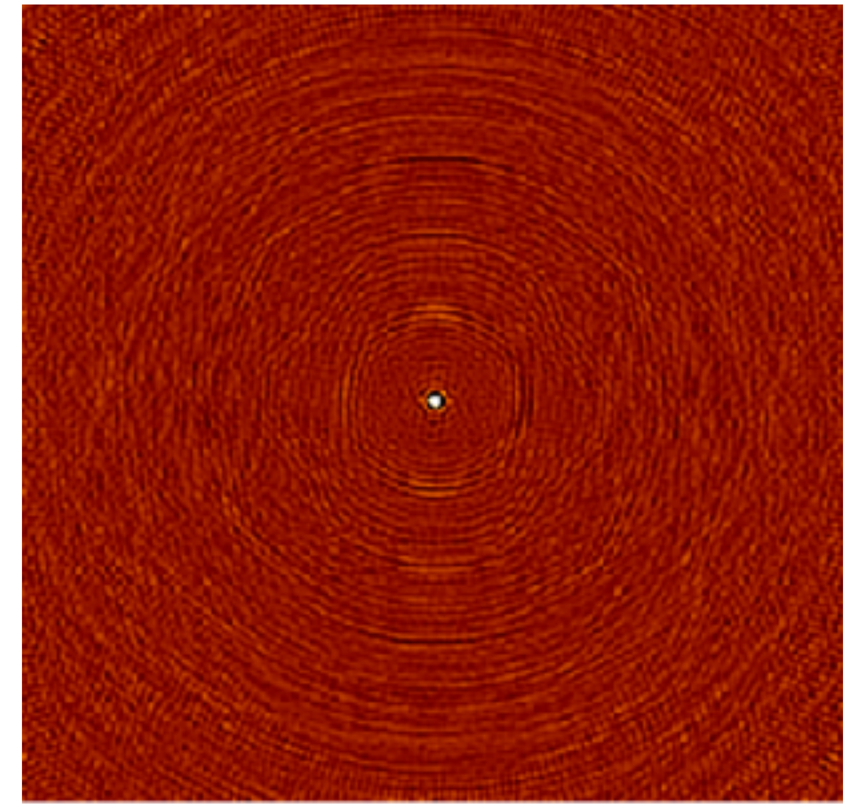


Some definitions

$V(u,v)$

Visibilities



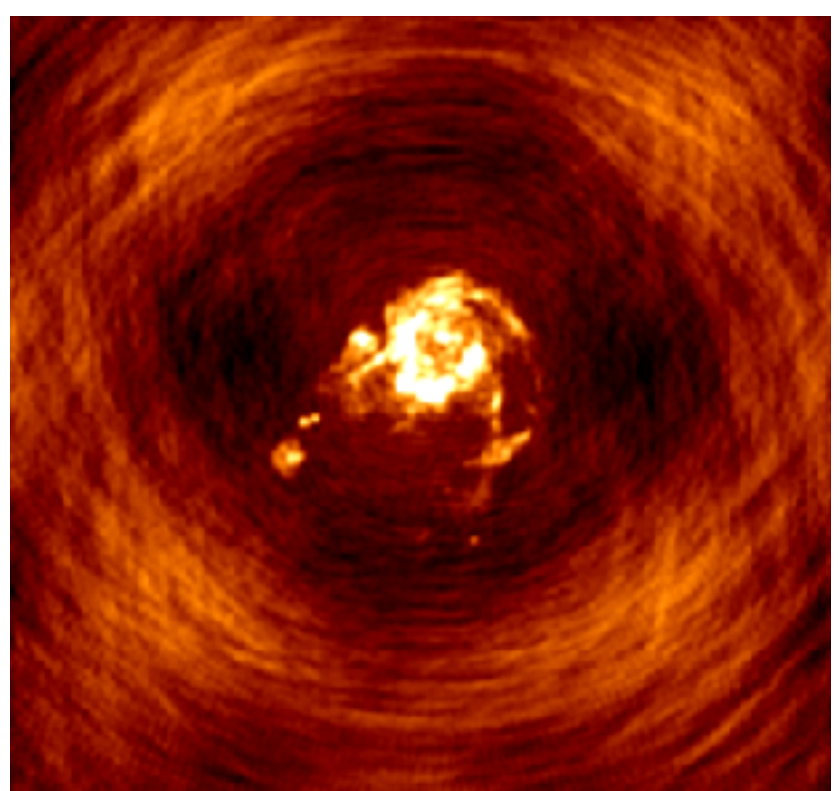


$B_D(x,y)$

Dirty beam

$I_D(x,y)$

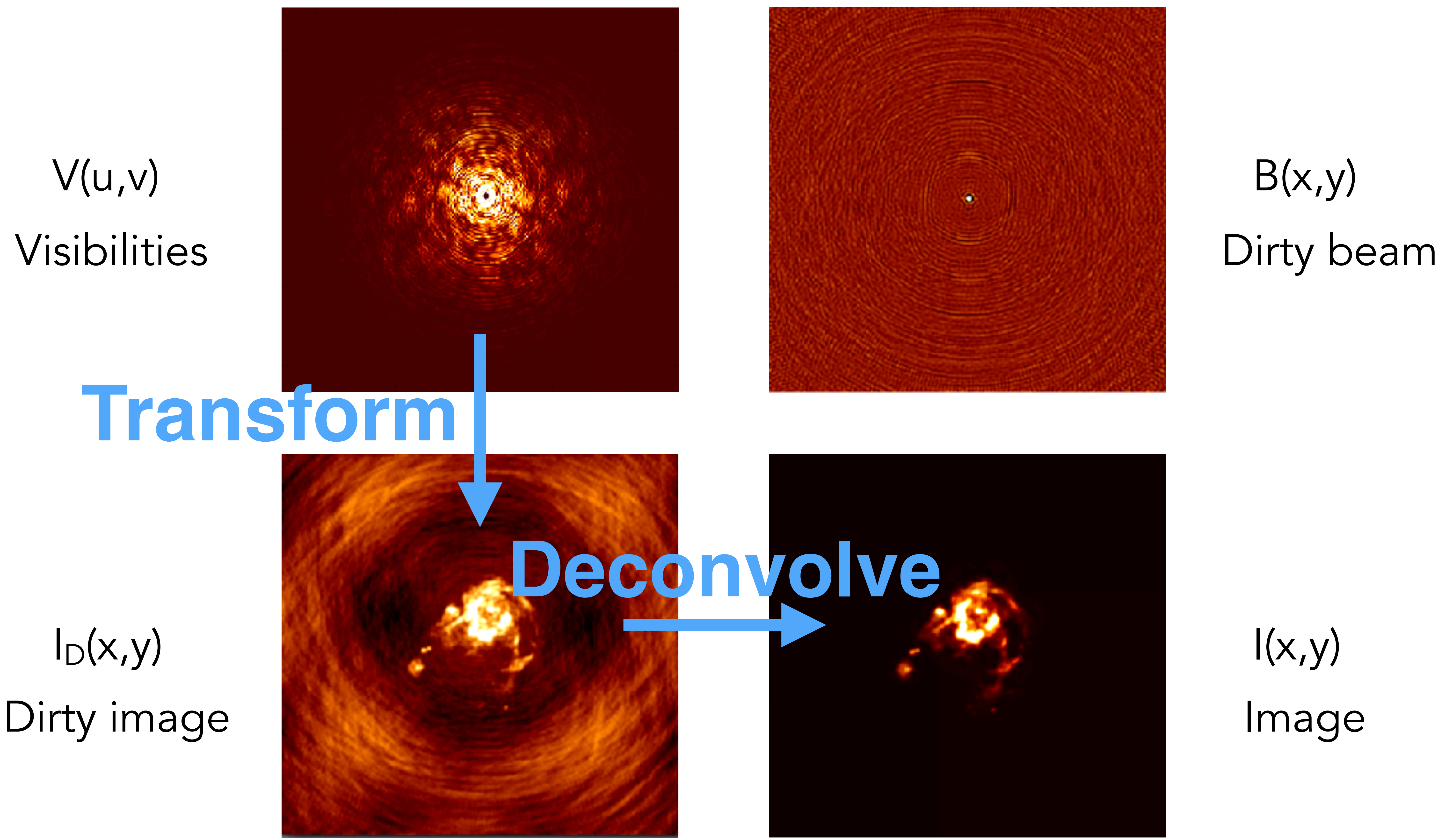
Dirty image



$I(x,y)$

Image

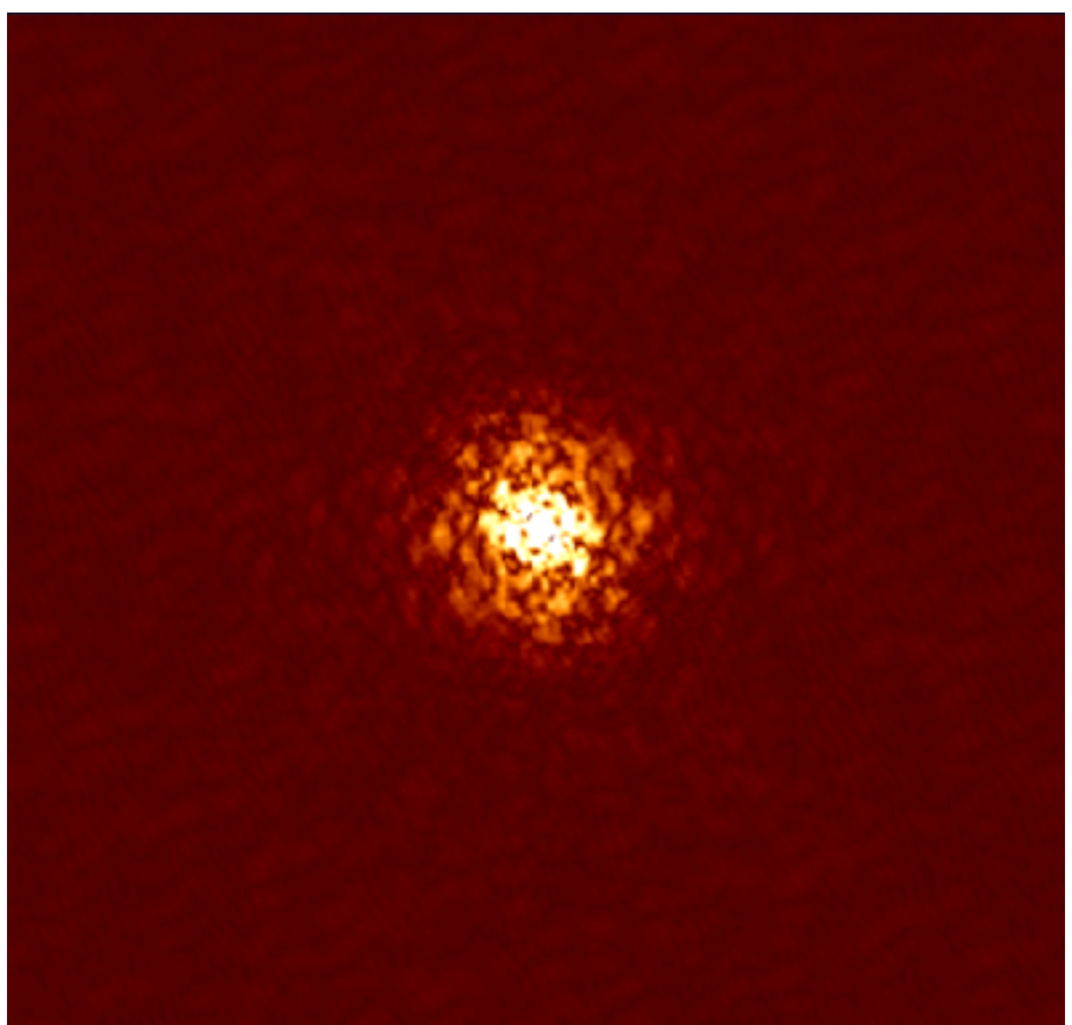
Nomenclature



Ideal Fourier transform

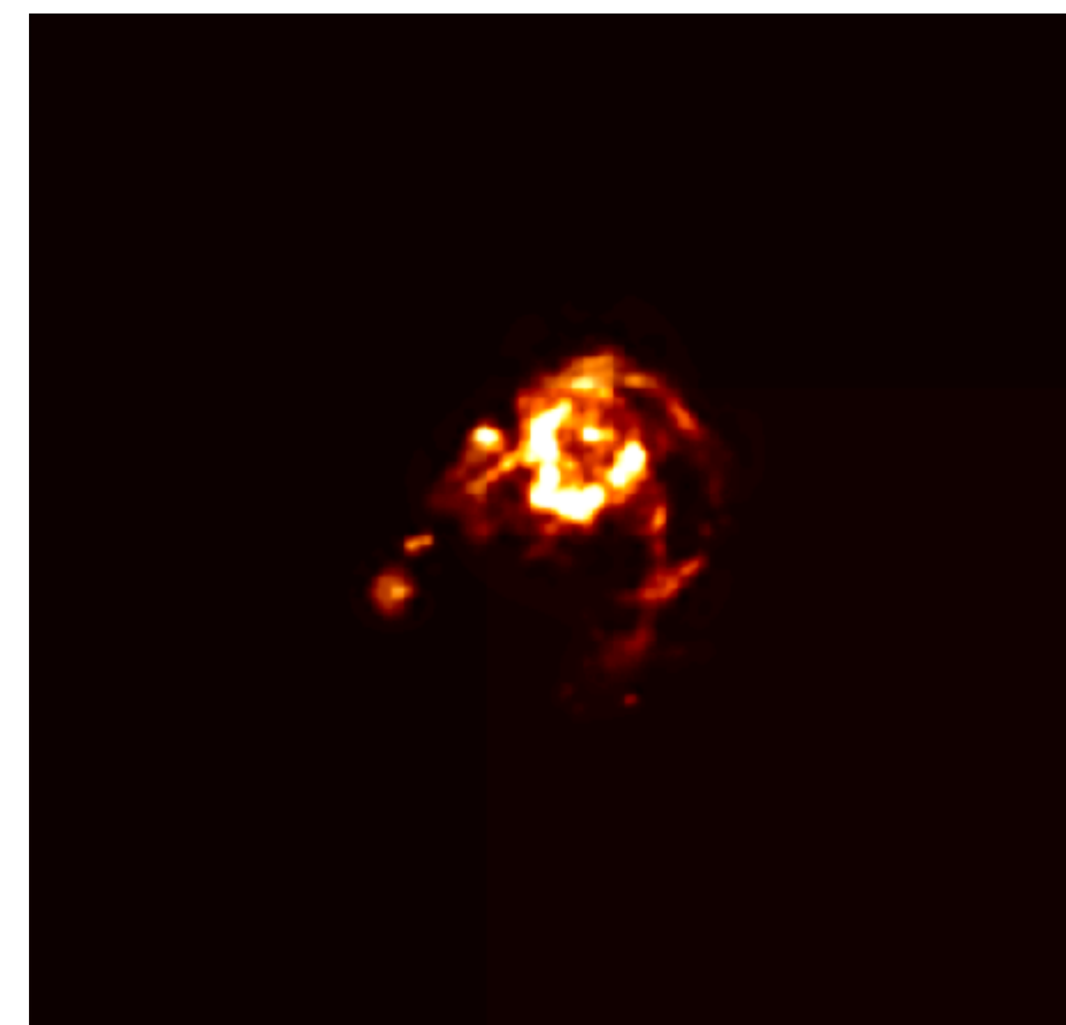
$$V(u,v) = \int I(x,y) e^{-2\pi i (ux+vy)} dxdy$$

Ideal visibilities



Fourier Transform

True image



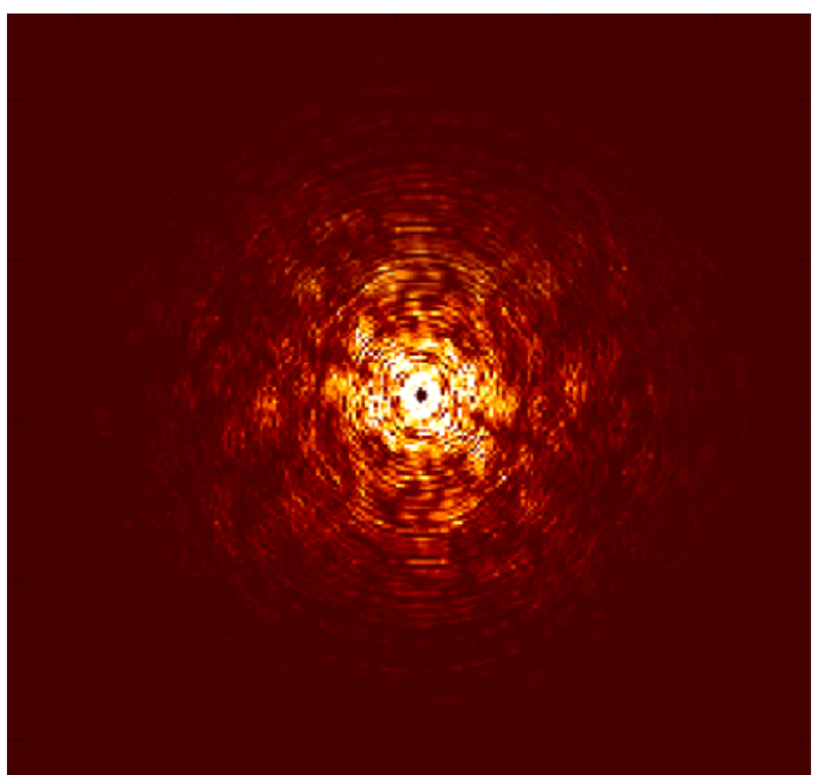
This is true if we measure V(u,v) in EVERY point!

Sampling of the (u,v) plane

With a limited number of antennas, the uv-plane (u,v) is sample in a discrete number of points

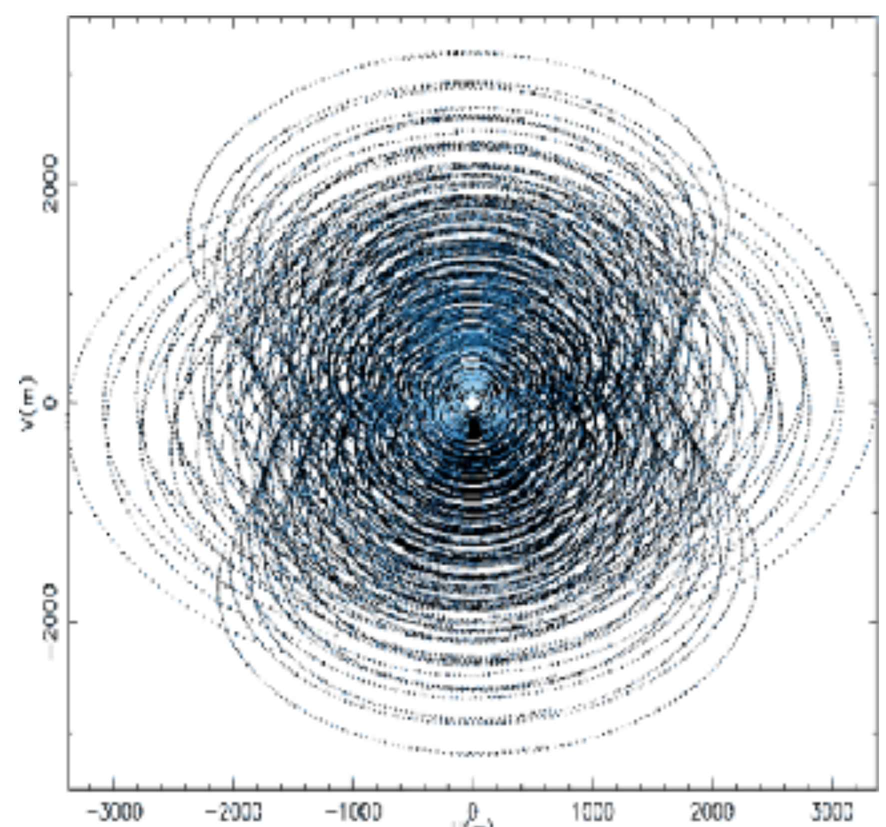
$$S(u,v) = \sum_k \delta(u_k, v_k) \qquad V_M(u,v) = S(u,v)V(u,v)$$

Measured



$V_M(u,v)$

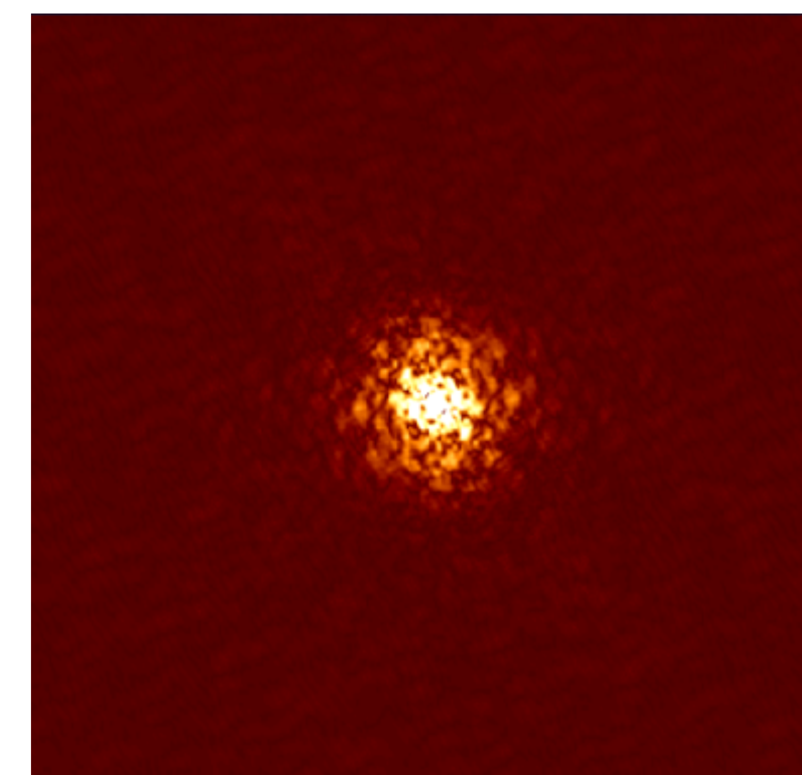
=



S(u,v)

x

Ideal



V(u,v)

Sampling of the (u,v) plane

The uv-plane is sampled in discrete points:
some information is lost

Outer limit

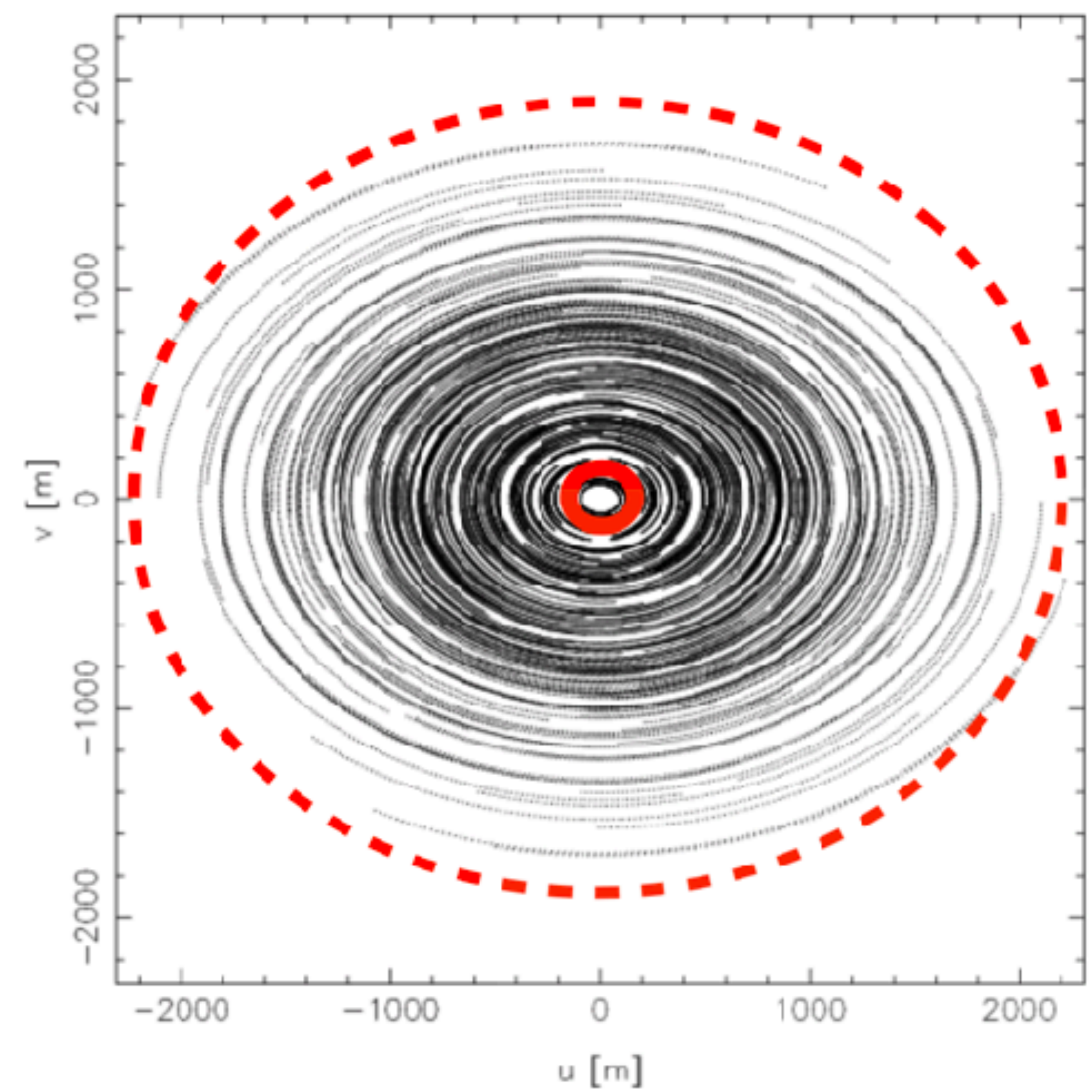
- We do not have measures beyond u_{max}, v_{max}
- It defines the angular resolution of the array

Inner limit

- There is a "hole" in inner regions of S(u,v)
- Information on large scales is missing
- Very extended structures are invisible ("spatial filtering")

Sparse sampling

- We miss information in several regions of the (u,v) plane
- It contributes to the structure of the side lobes



Sampling of the (u,v) plane

The Fourier transform of the measured visibilities gives the dirty image:

$$I_D(x,y) = FT^{-1}[V_M(u,v)] = FT^{-1}[S(u,v)V(u,v)]$$

Using the convolution theorem we obtain:

$$I_D(x,y) = B_D(x,y) * I(x,y) \qquad B_D(x,y) = FT^{-}1[S(u,v)]$$

The dirty image is the convolution of the dirty beam with the true image

To obtain I(x,y) we need to deconvolve B_D(x,y) from I_D(x,y)

"Weighting schemes"

The dirty image (and dirty beam) can be weighted:

$$I_D(x, y) = \frac{\sum_k FT^{-}1[W_k(u, v)S(u, v)V(u, v)]}{\sum_k W_k}$$

$$W_k = \frac{1}{\sigma_k^2}$$

Natural

- Maximises point source sensitivity, high side-lobes

$$W_k = \frac{1}{\sigma_k^2 \rho(u_k, v_k)}$$

Uniform

- High angular resolution, lower sensitivity to point sources.

$$W_k = \frac{1+s}{\sigma_k^2[1 + \frac{s\rho(u_k, v_k)}{\sigma_k^2}]}$$

Robust

- It's an intermediate choice between natural and uniform, the parameter s ranges [-2,2]

$$W_k = \frac{1}{\sigma_k^2} e^{-\frac{(u^2+v^2)}{t^2}}$$

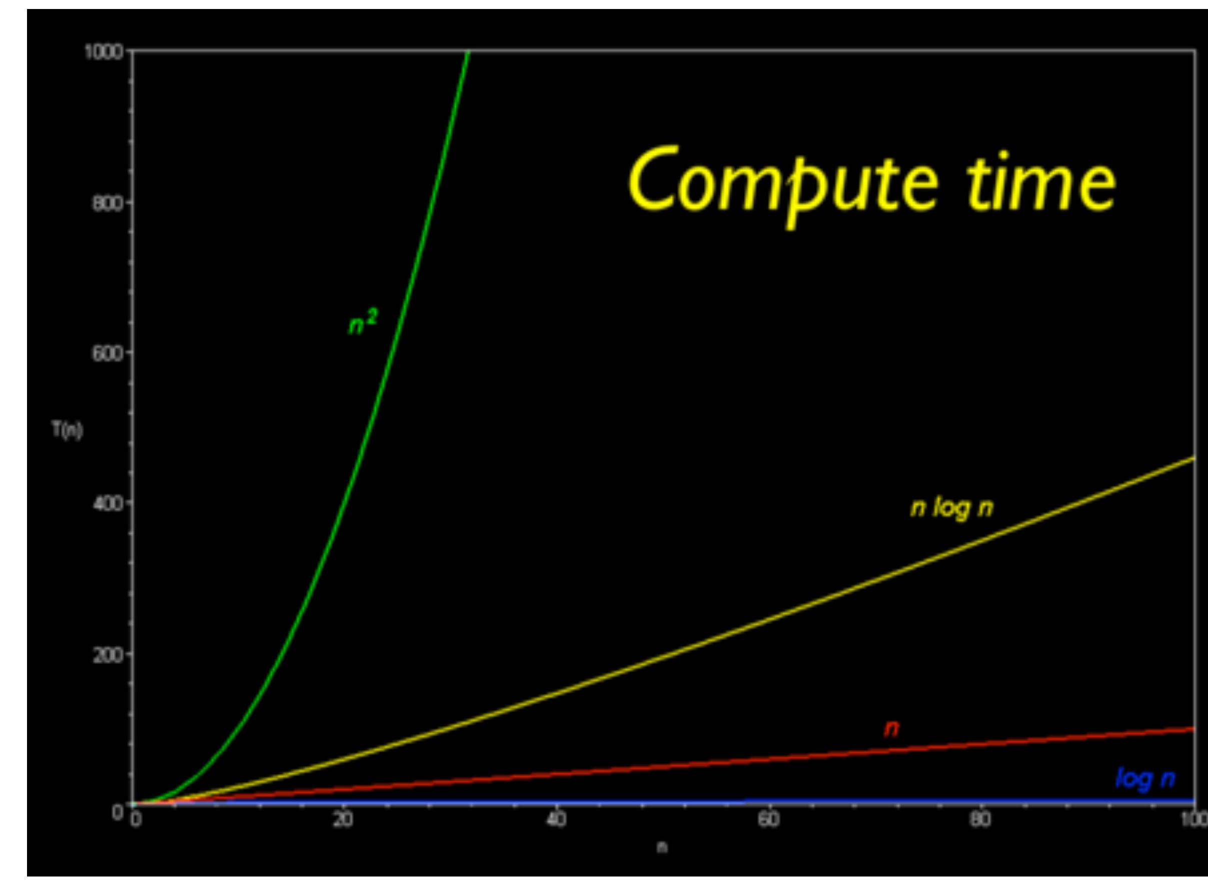
Tapering

- Adds a down-weighting to the high-spatial frequencies

Computing the dirty image

Fourier Transform

- Use algorithm Fast Fourier Transform (FFT)
- Computing time scales with *N*log*N* for a *N* x *N* image
- FFT needs a regular grid

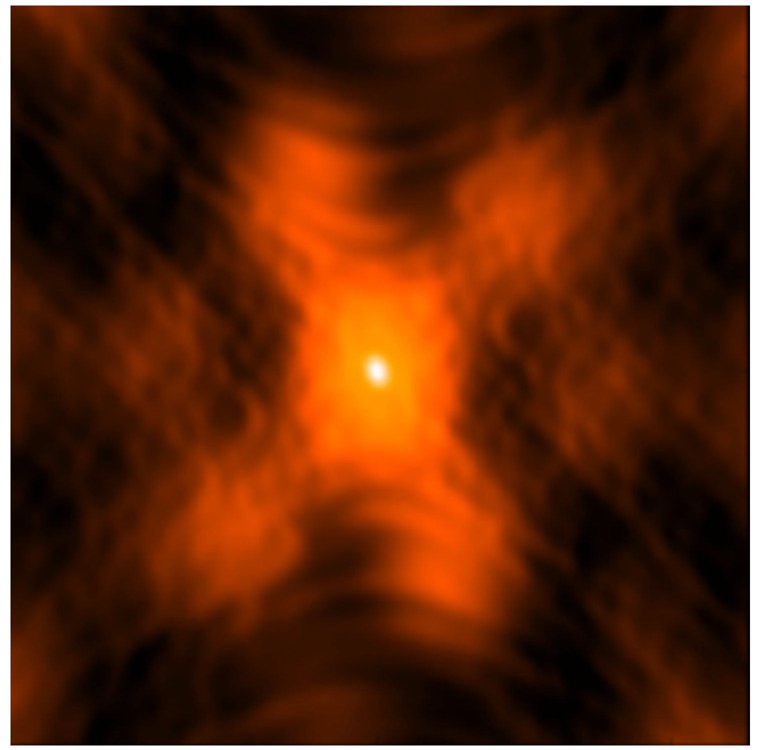


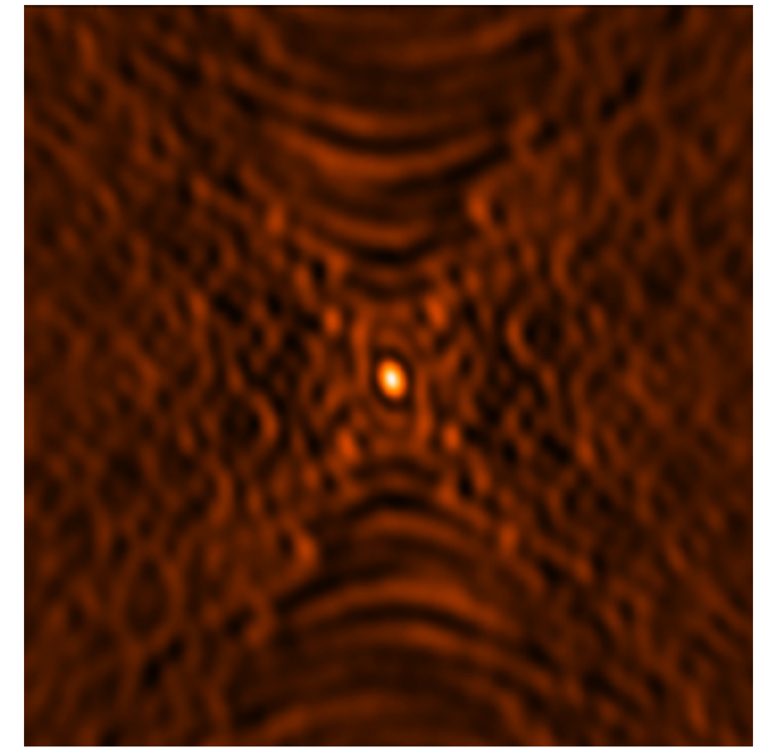
Gridding

- The data $V_M(u,v)$ are obtained on an irregular grid
- They are mapped onto a regular grid (sides are a multiple of 2)
- Grid points where no uv-data are present are set to 0.

“Weighting schemes”

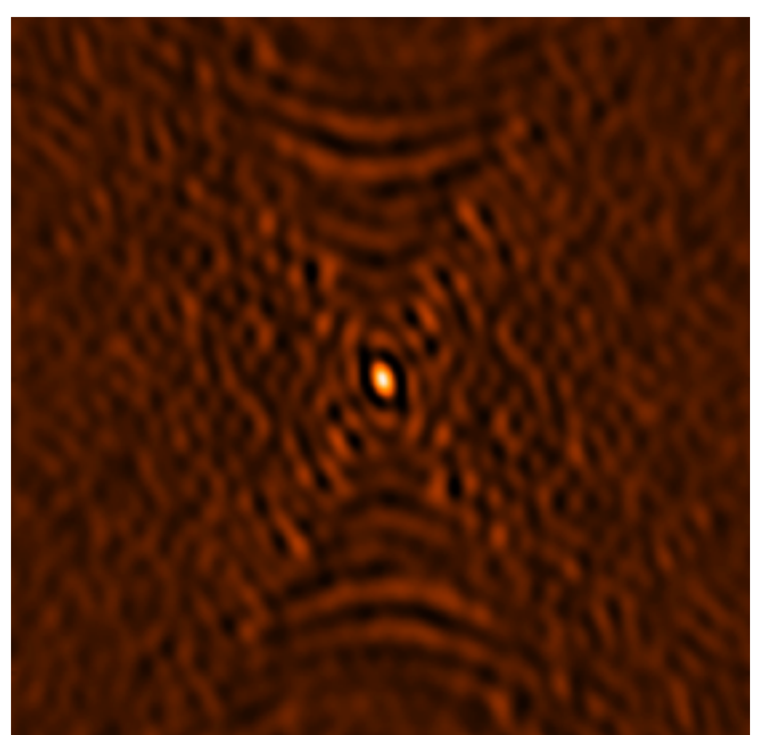
Natural
Beam ~ 0.77 x 0.62

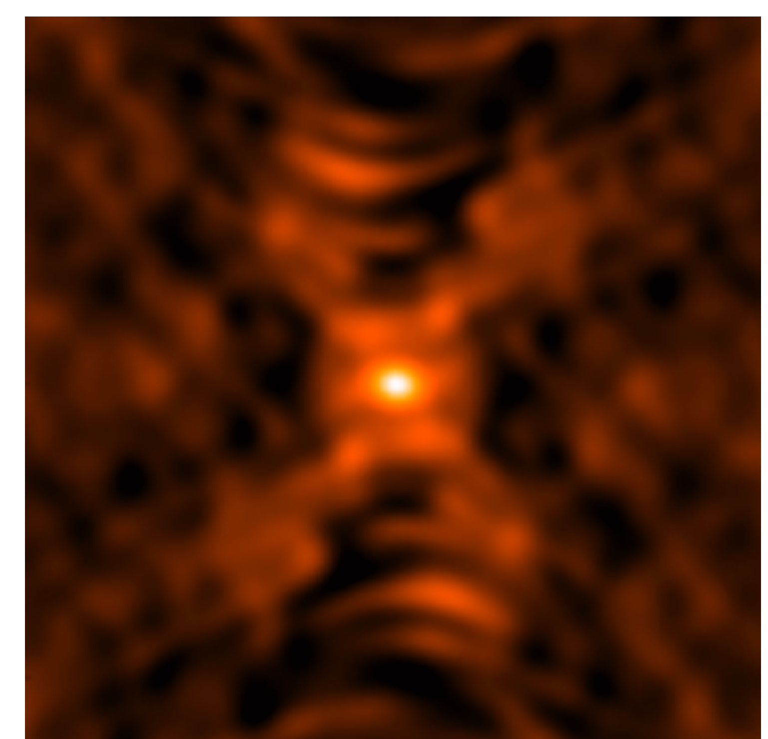




Robust -1
Beam ~ 0.41 x 0.36

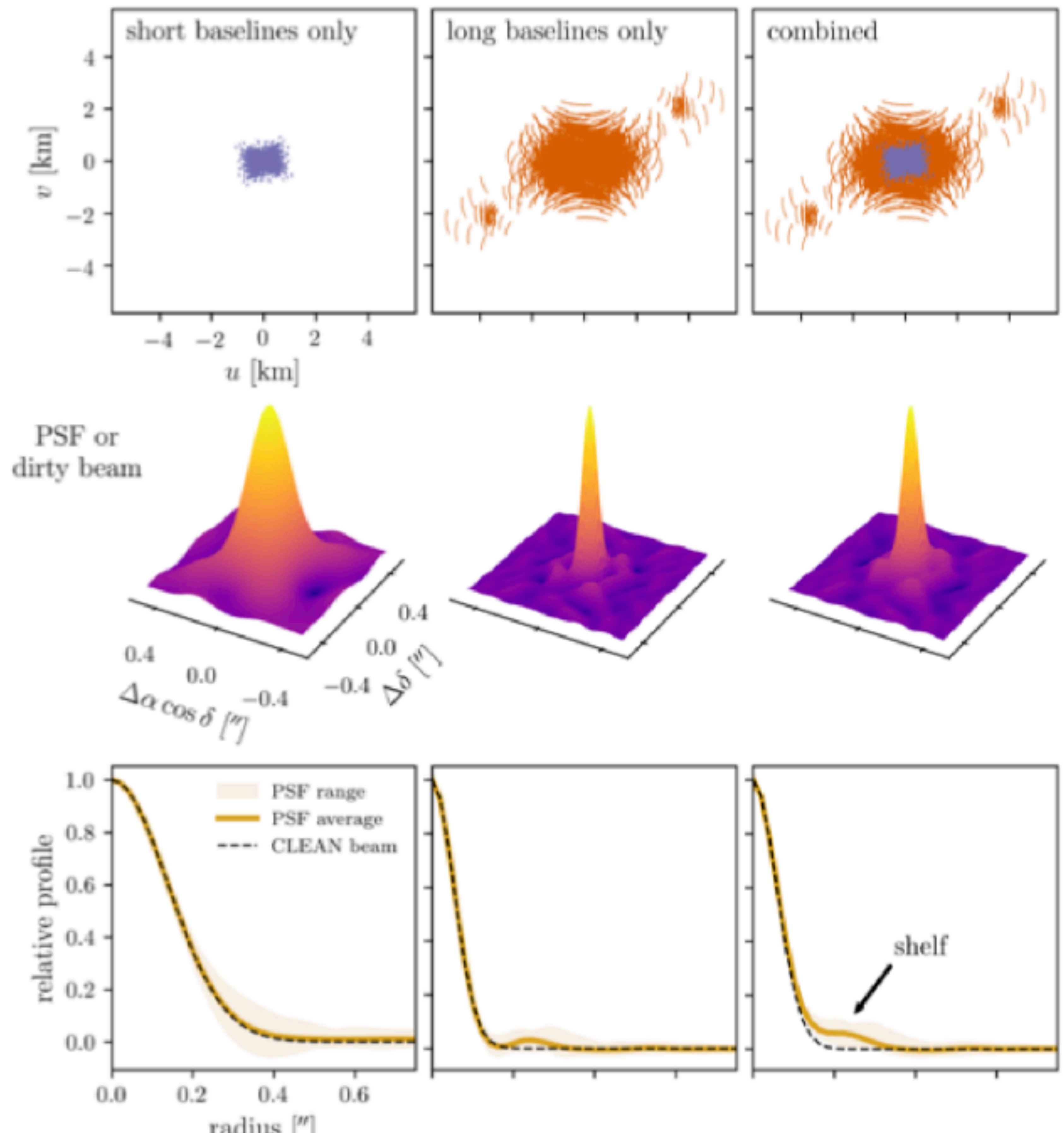
Uniform
Beam ~ 0.39 x 0.31





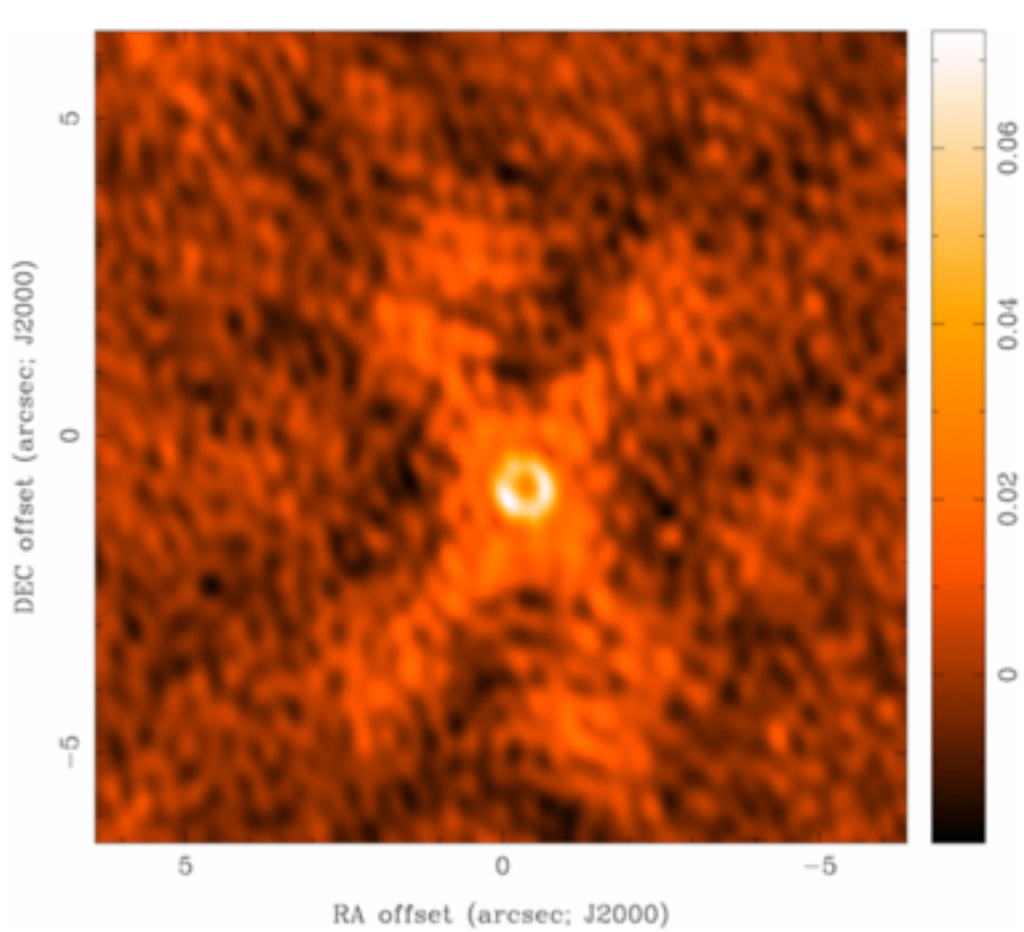
uv-taper
Beam ~ 0.77 x 0.62

Dirty beam combining different configurations

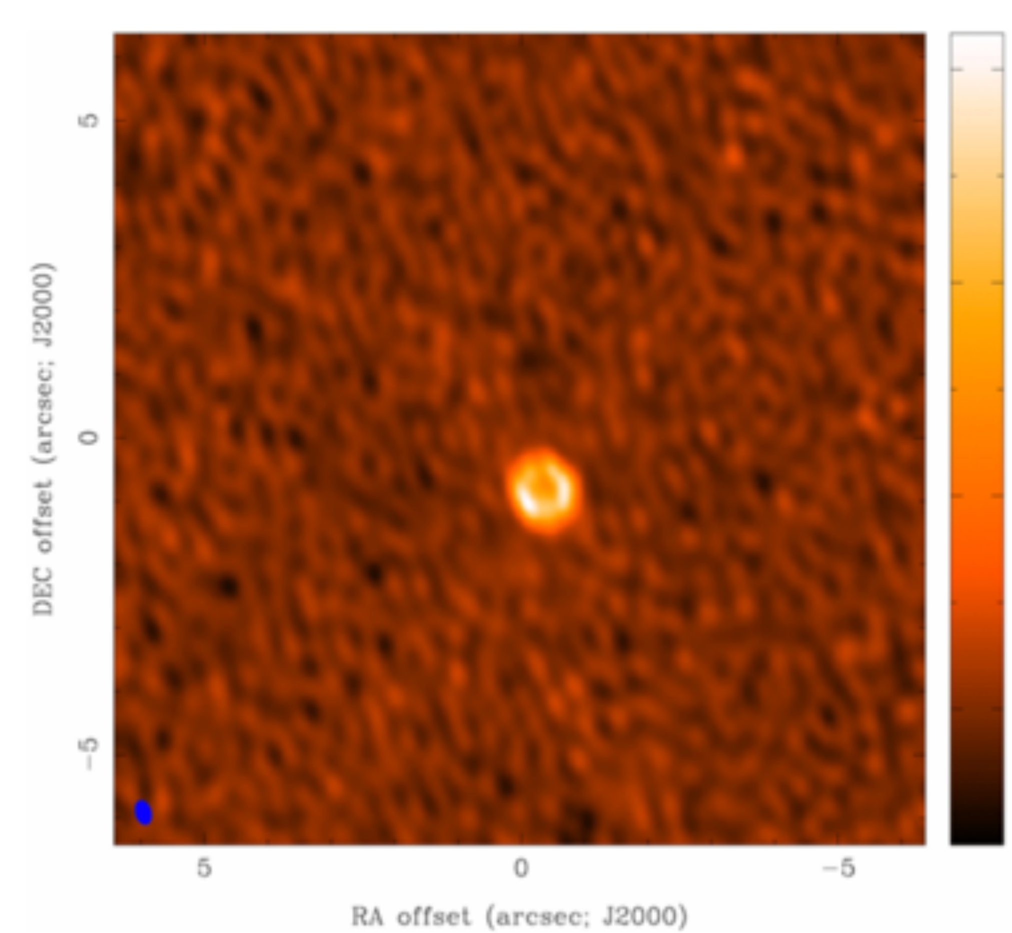


Deconvolution

- After calibration and FT: $V_M(u,v)$ -> $I_D(x,y)$
- We need to deconvolve $B_D(x,y)$ from $I_D(x,y)$ to obtain $I(x,y)$
- Information is incomplete, there is noise in the data: pay attention with this step!



Dirty image



Scientific image

Methods of image reconstruction

- **CLEAN**
  The sky model is the sum of point-sources
- **Multi-scale CLEAN** (very relevant for disks)
  The sky model is the linear combination of Gaussians with FWHM defined by the user (typically 5)
- **Adaptive-Scale-Pixel CLEAN** (may be used more in near future)
  The sky model is the linear combination of Gaussians with FWHMs that are computed on the fly.
- **Regularized Maximum Likelihood (RML) algorithms**
  This is not a deconvolution process, but it's a non-parametric forward model of the sky intensity, with additional constraints (e.g., smoothness, positivity).

CLEAN

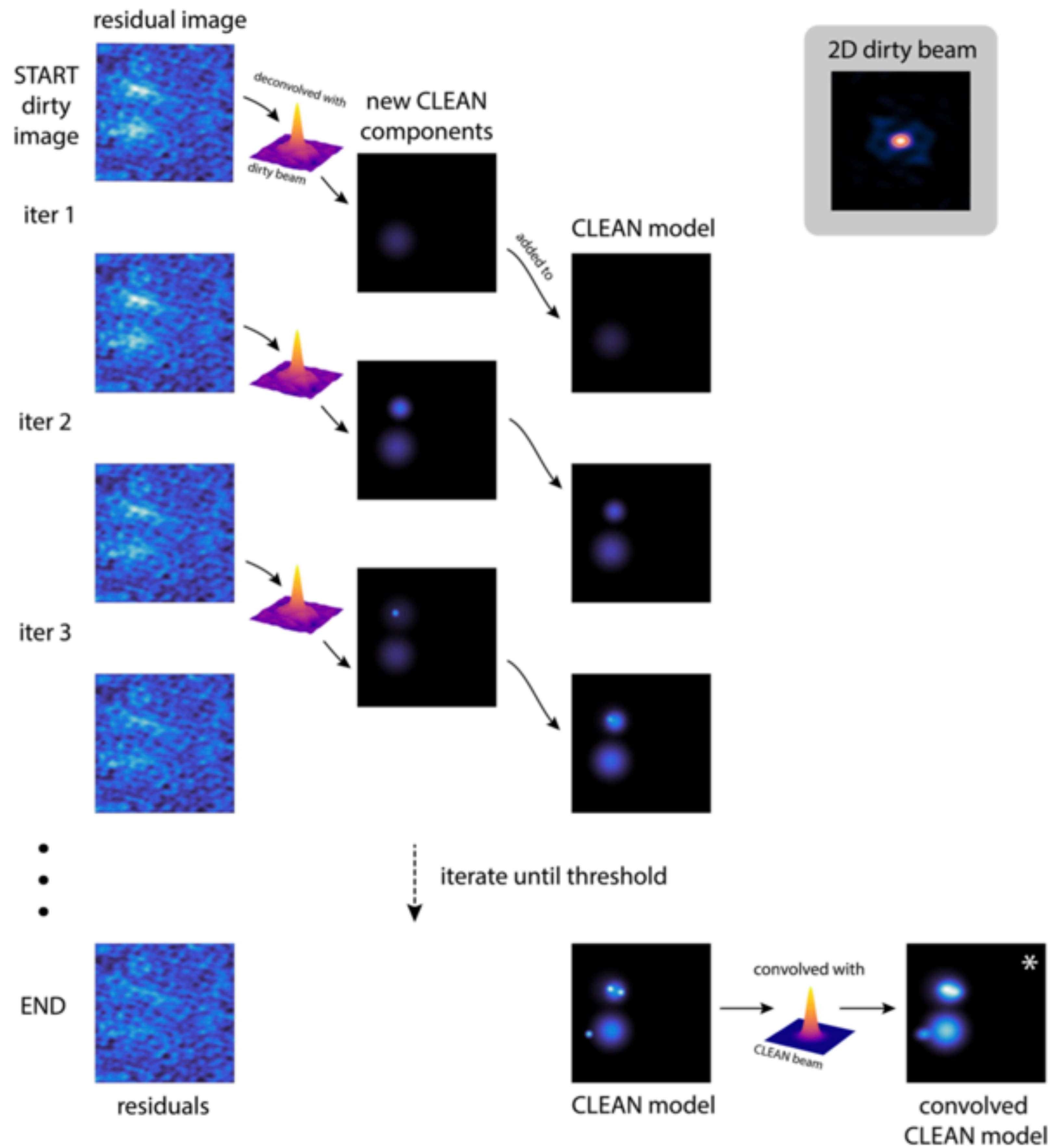
Developed by Hogbom in 1974. It assumes that the sky is a the linear combination of point sources:

$$I(x, y) = \sum_i a_i \delta(x_i, y_i)$$

Iterative process:

1. The dirty beam and dirty image are computed with FFT.
2. We find the intensity peaks
3. We assign a delta function to the position of the peak with the measured amplitude to a sky model
4. We subtract a fraction of the model from the residual map using a parameter gain γ
5. We go back to step 2 and keep going, until we reach a threshold we had defined for the maximum residual
6. We convolve the sky model with a Gaussian fit of the primary beam and we add the residual map to obtain the final image

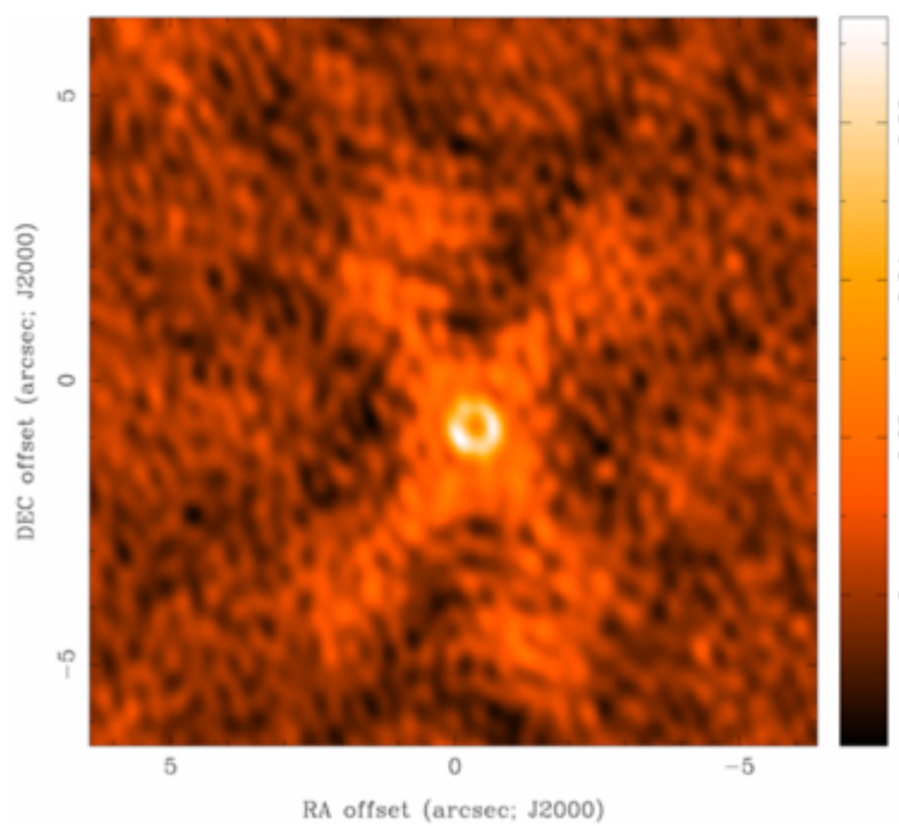
CLEAN



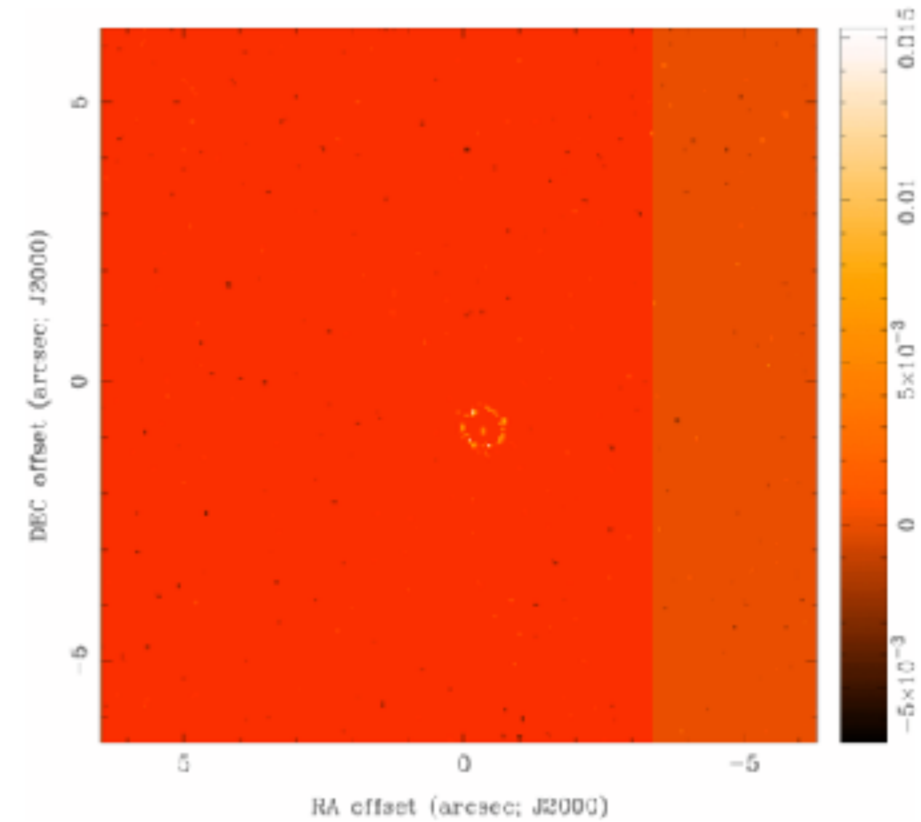
residual image
START
dirty
image
deconvolved with
dirty beam
new CLEAN
components
2D dirty beam
iter 1
added to
CLEAN model
iter 2
iter 3
iterate until threshold
END
convolved with
CLEAN beam
residuals
CLEAN model
convolved
CLEAN model

CLEAN

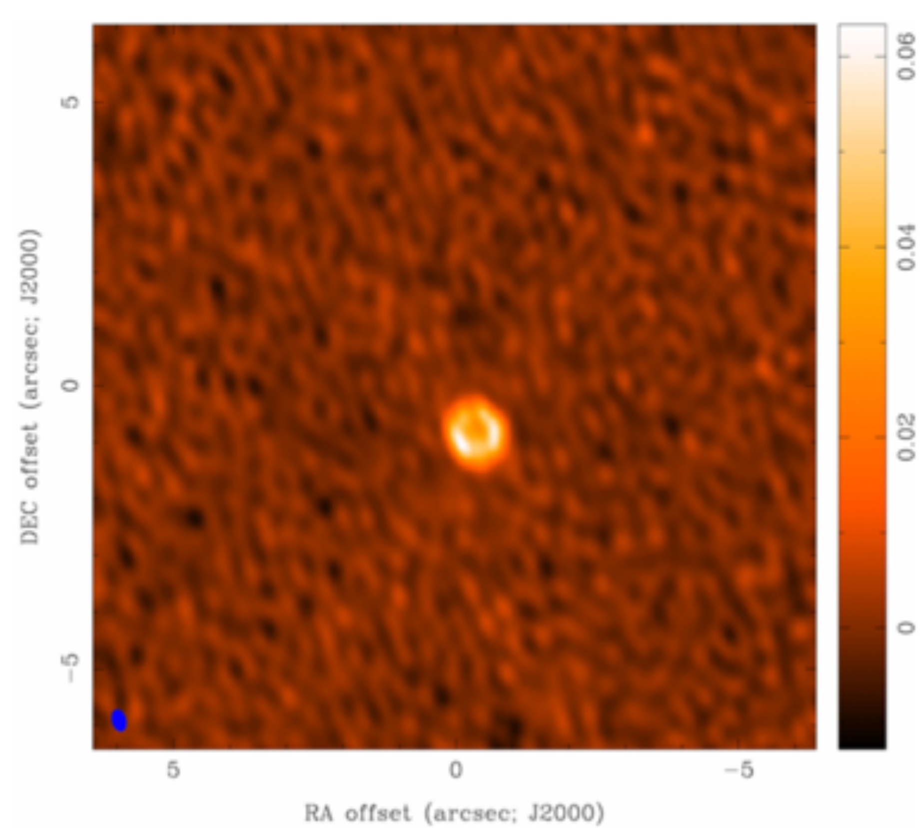
Dirty image



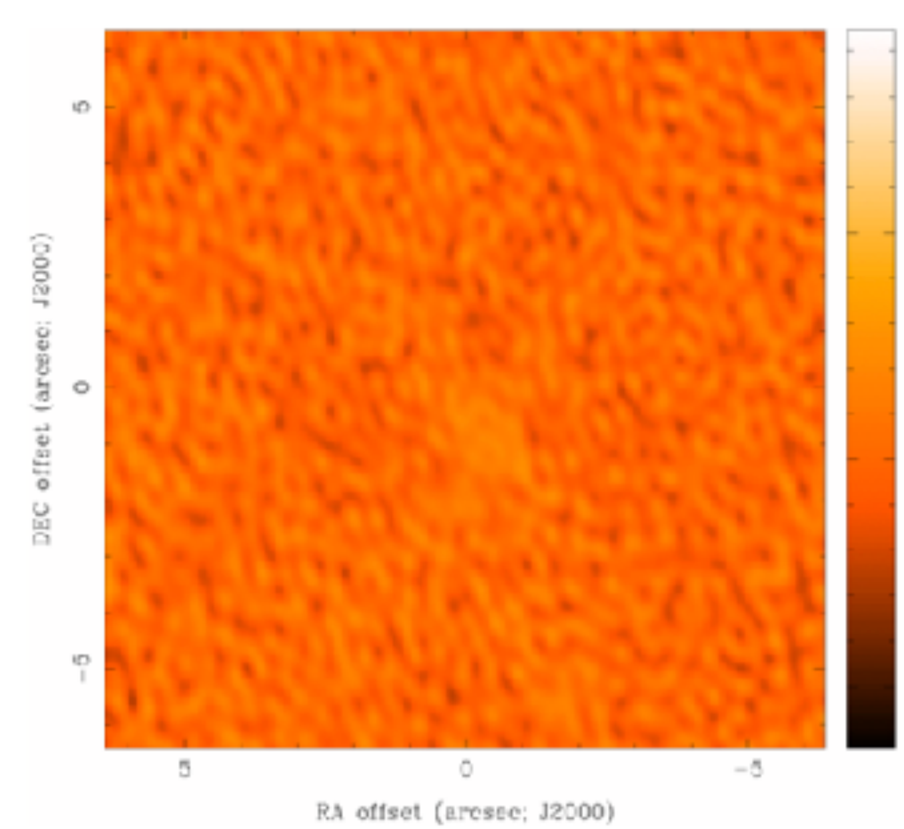
CLEAN model



Final image

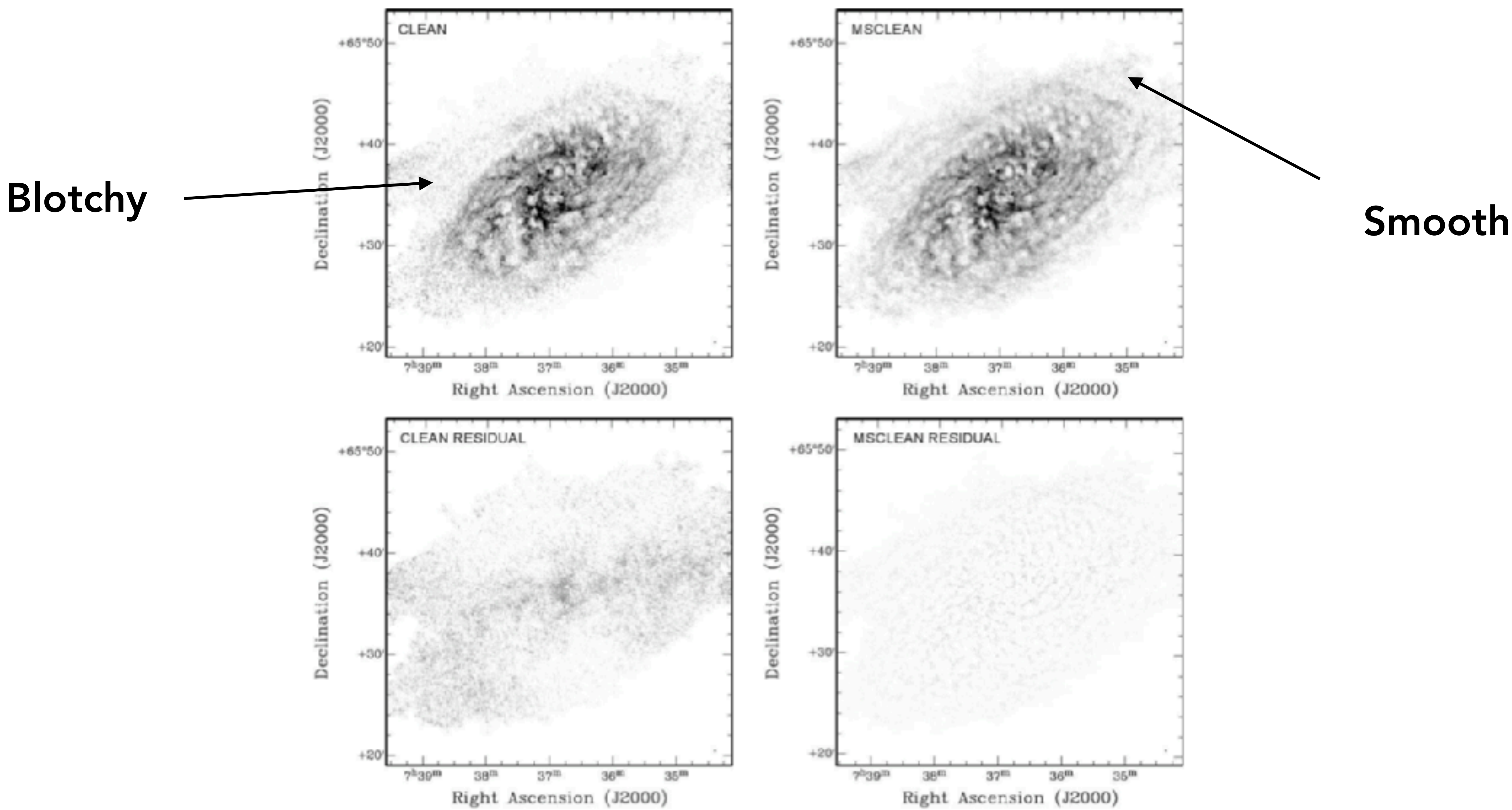


Residual map



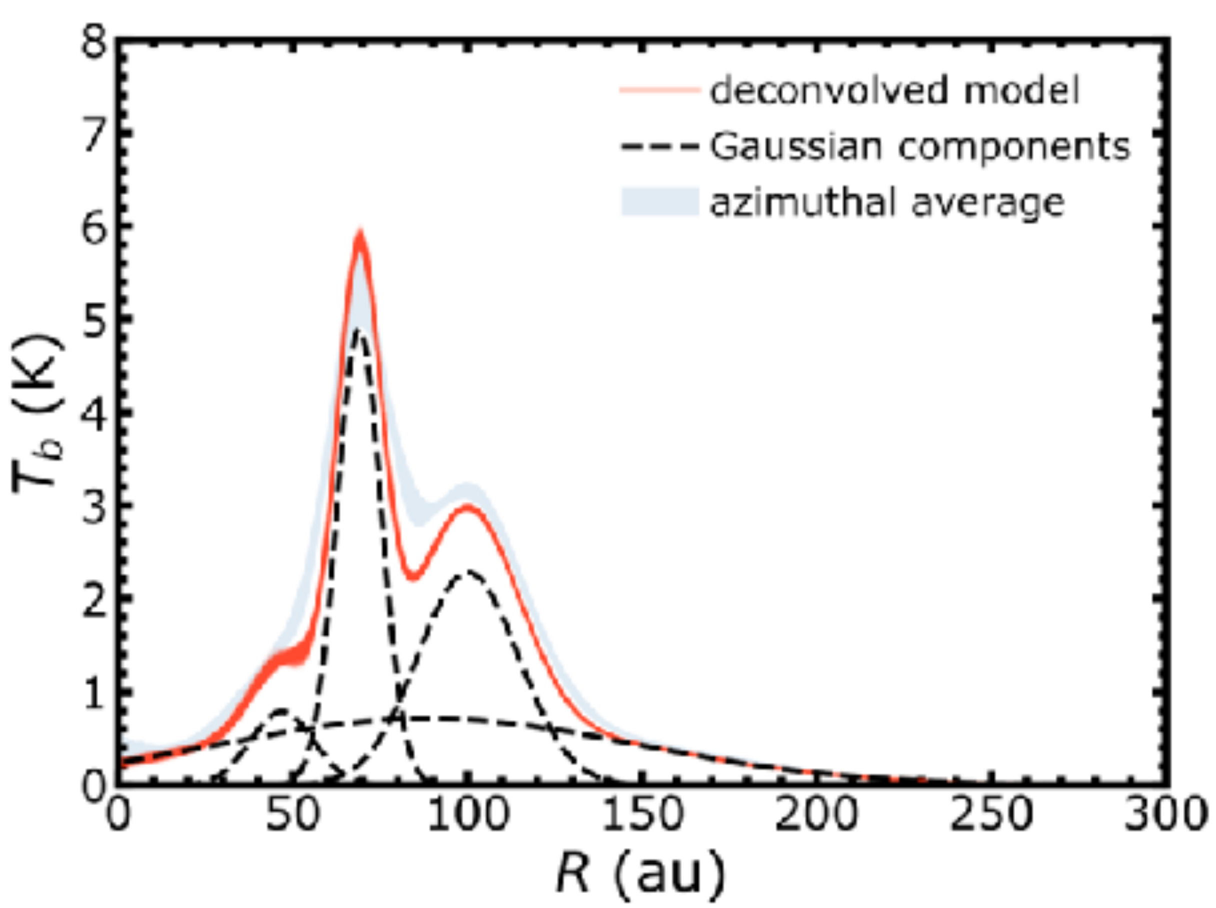
Multi-scale CLEAN

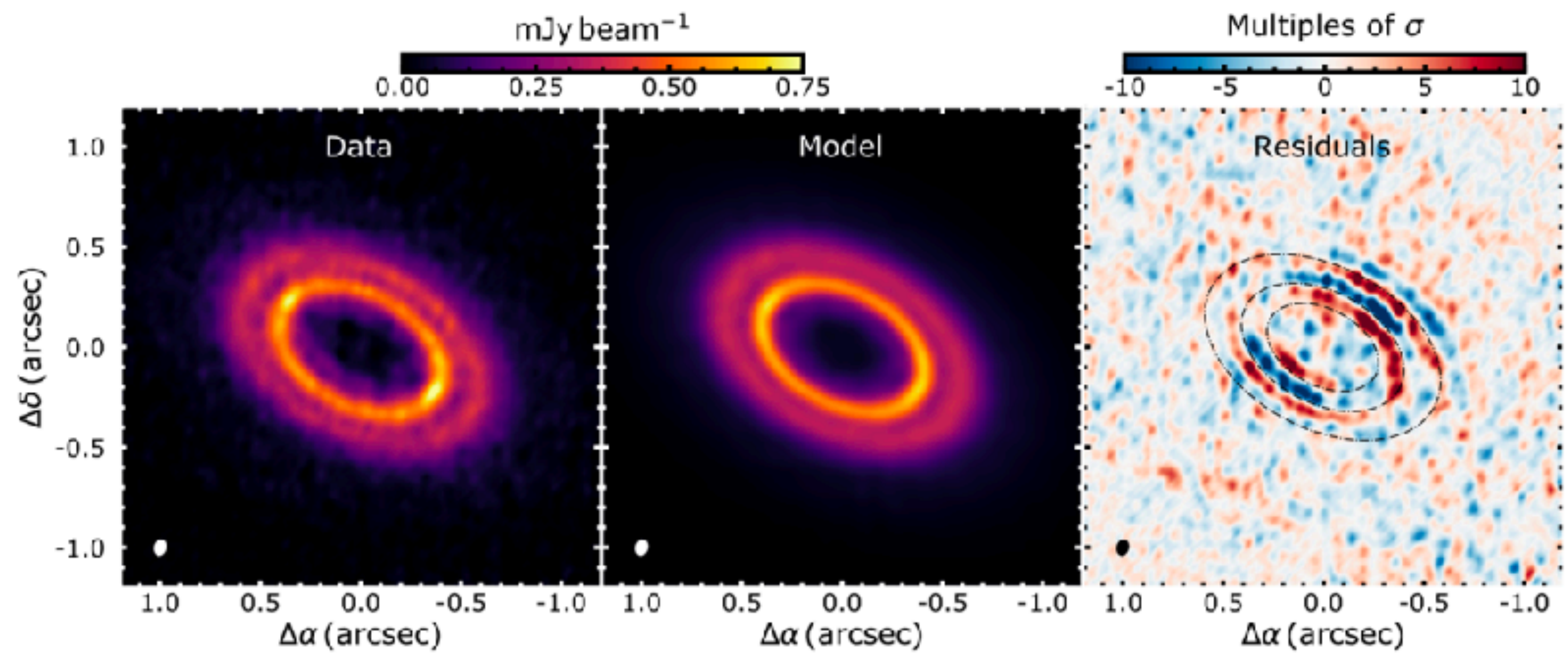
It is very relevant for extended sources
(classic CLEAN is optimized for point sources)



Data analysis in the uv-plane

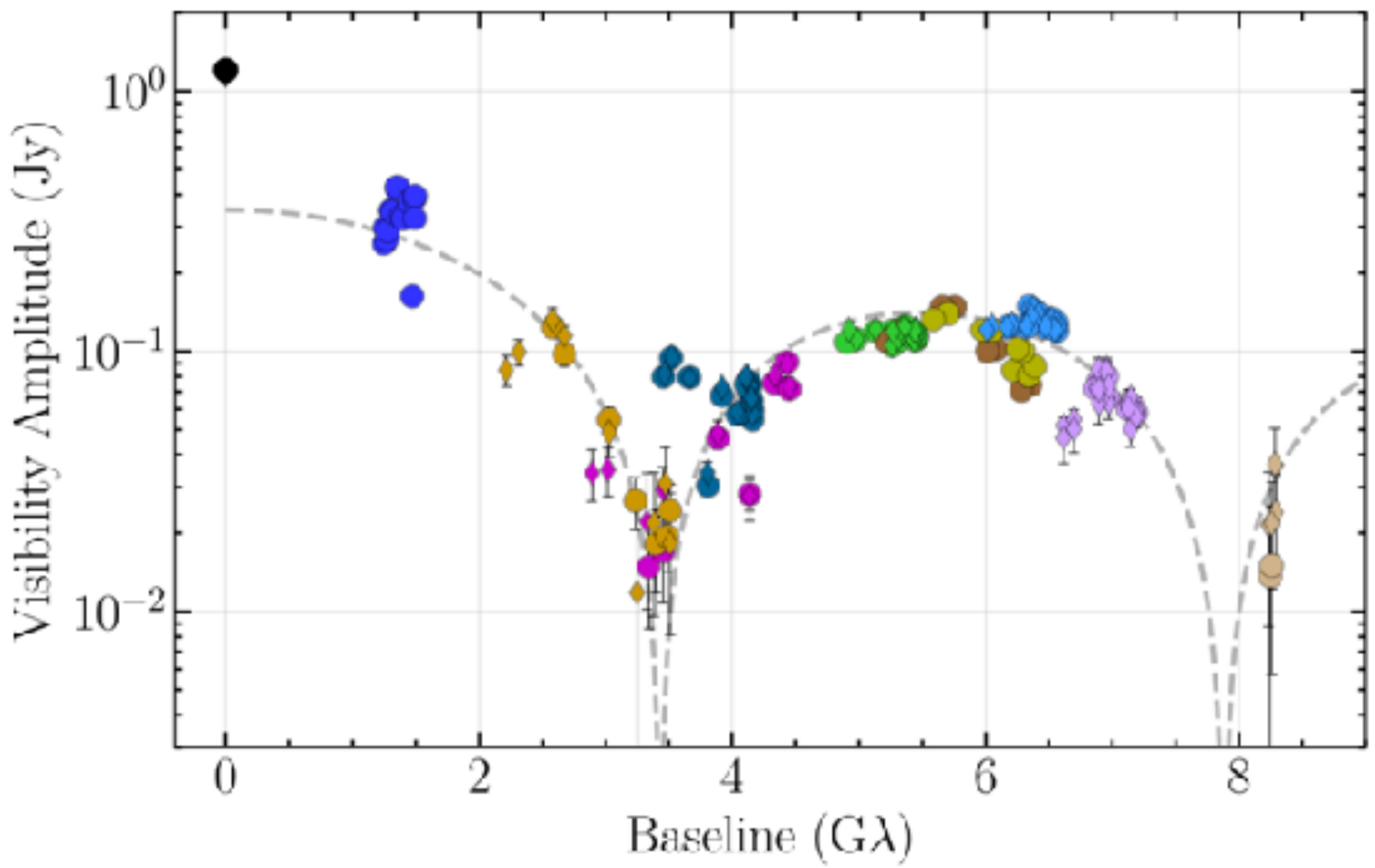
In order to avoid the convoluted error propagation in the image reconstruction, in several cases the data are fitted directly in the uv-plane

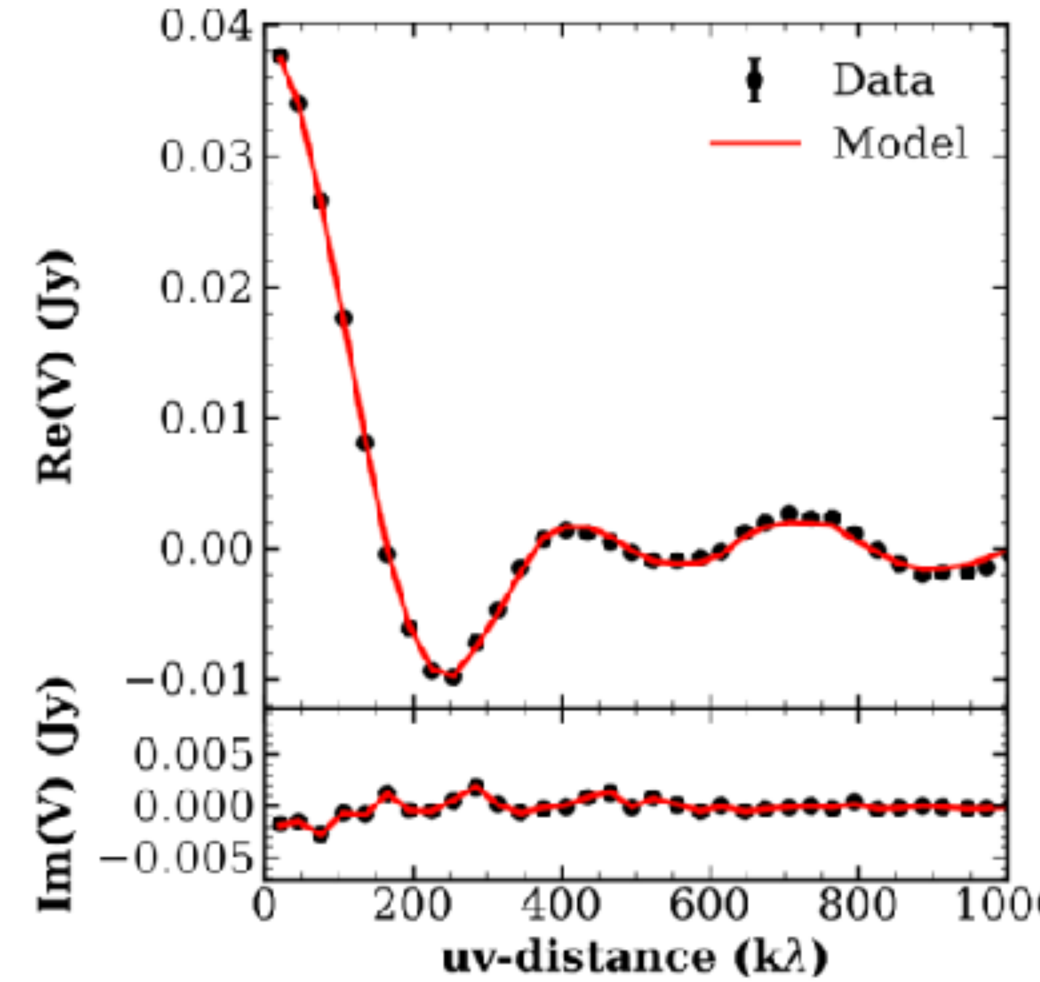


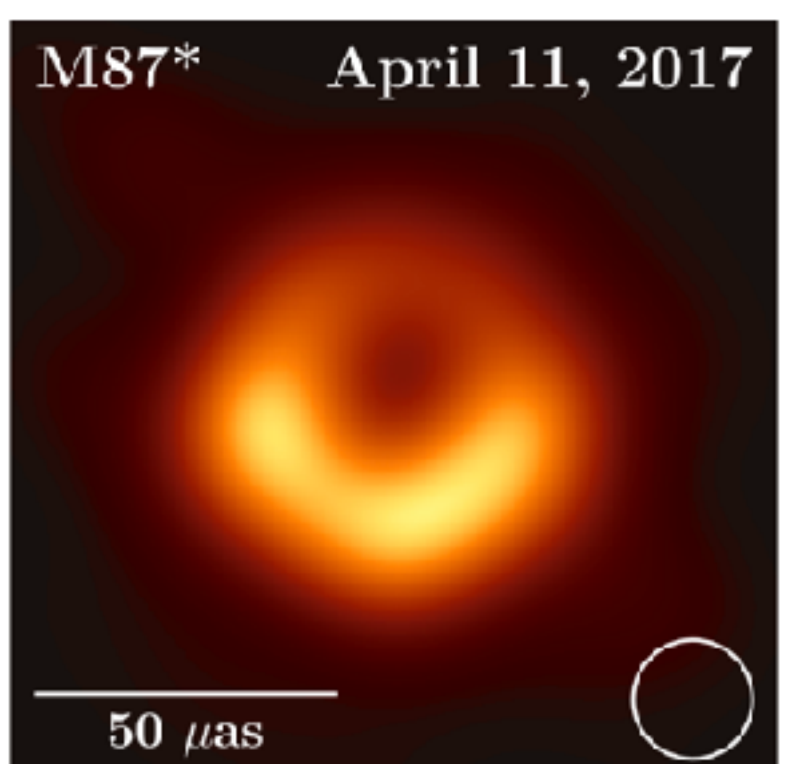


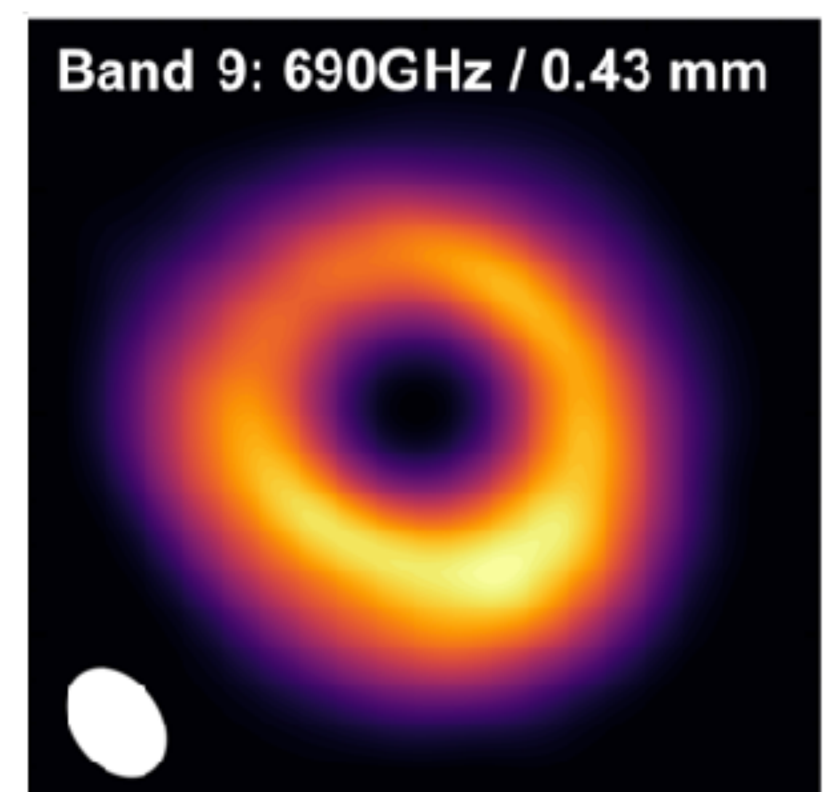
- Better treatment of the uncertainties
- It is possible to make better use of the very long baselines to reach a "super"-resolution

Data analysis in the uv-plane

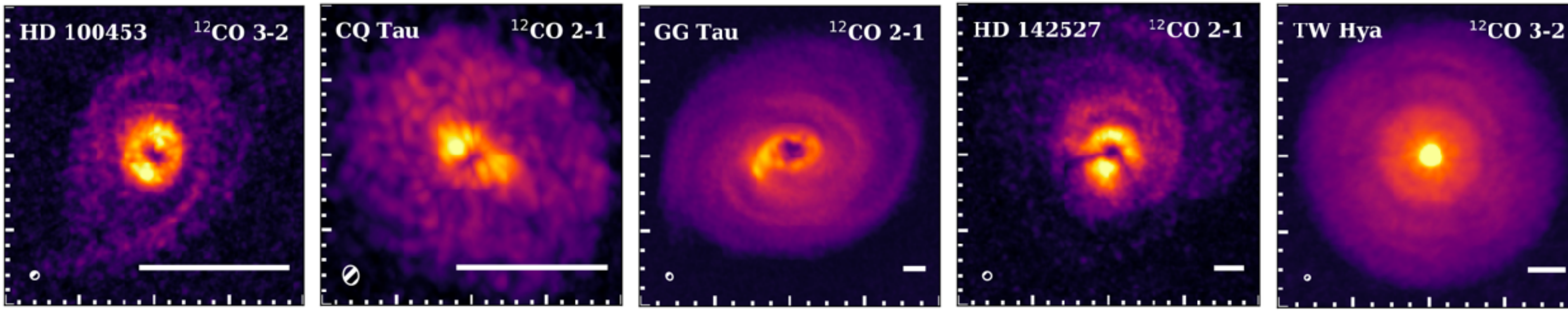








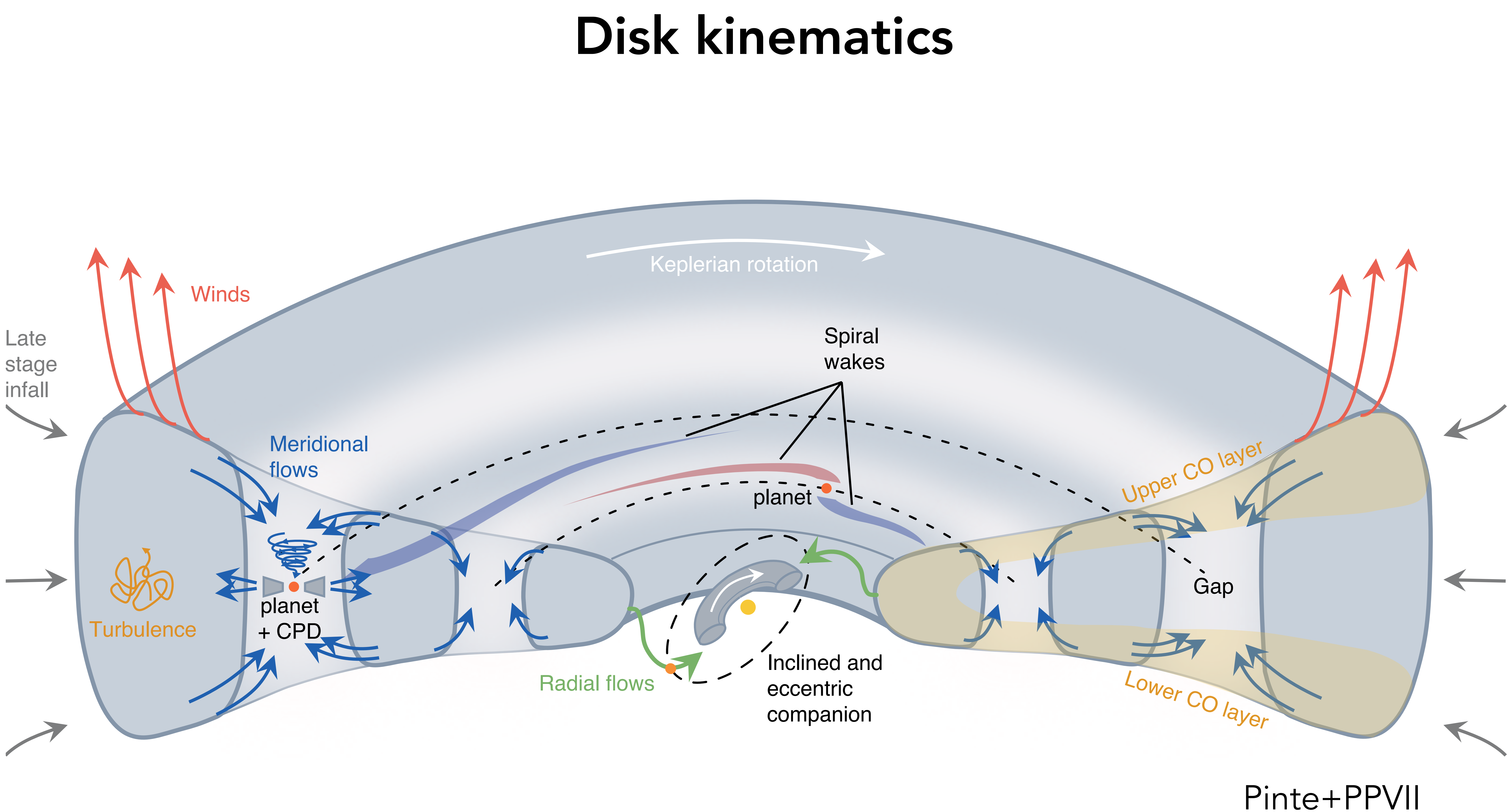
Disk kinematics

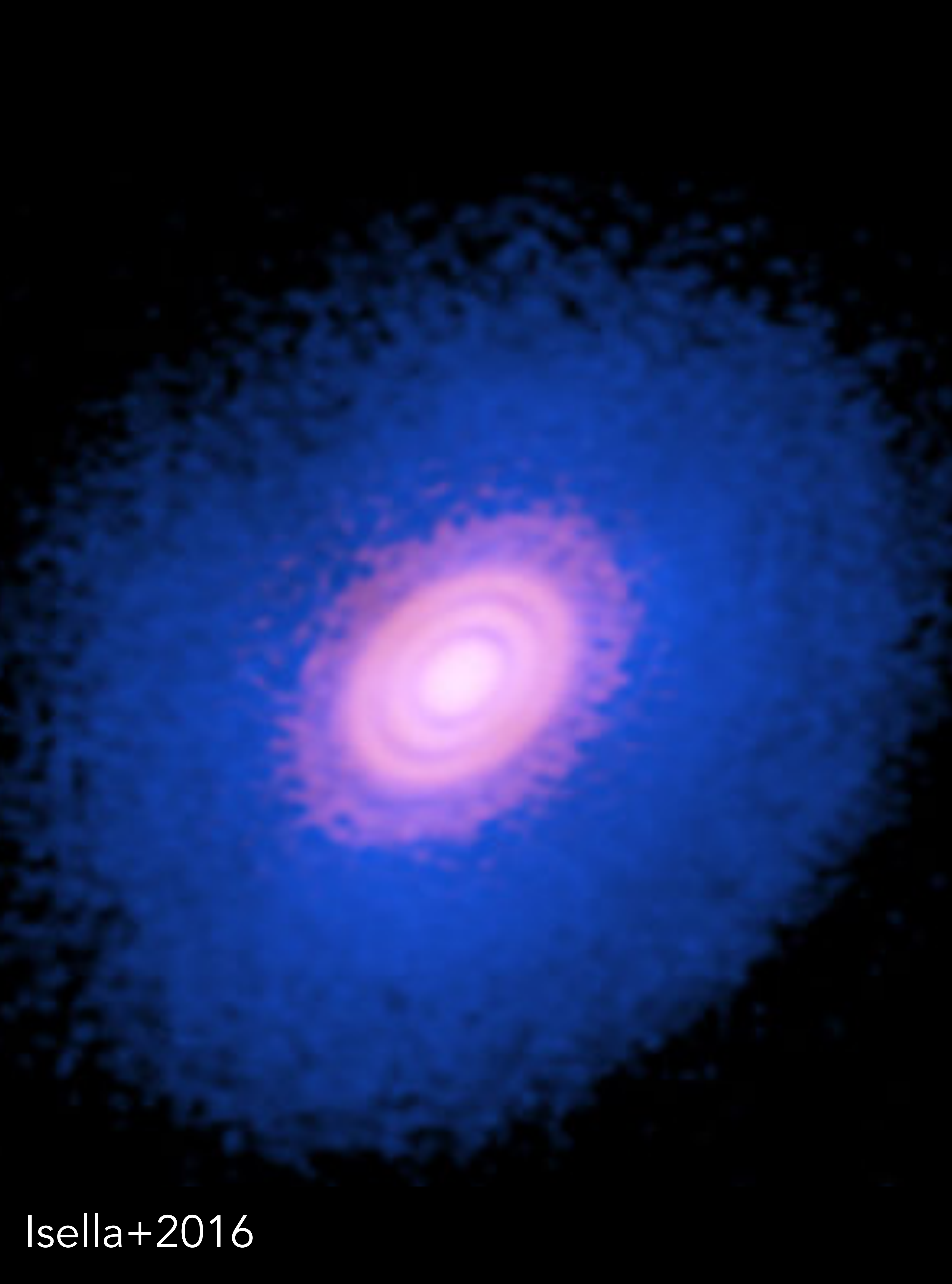


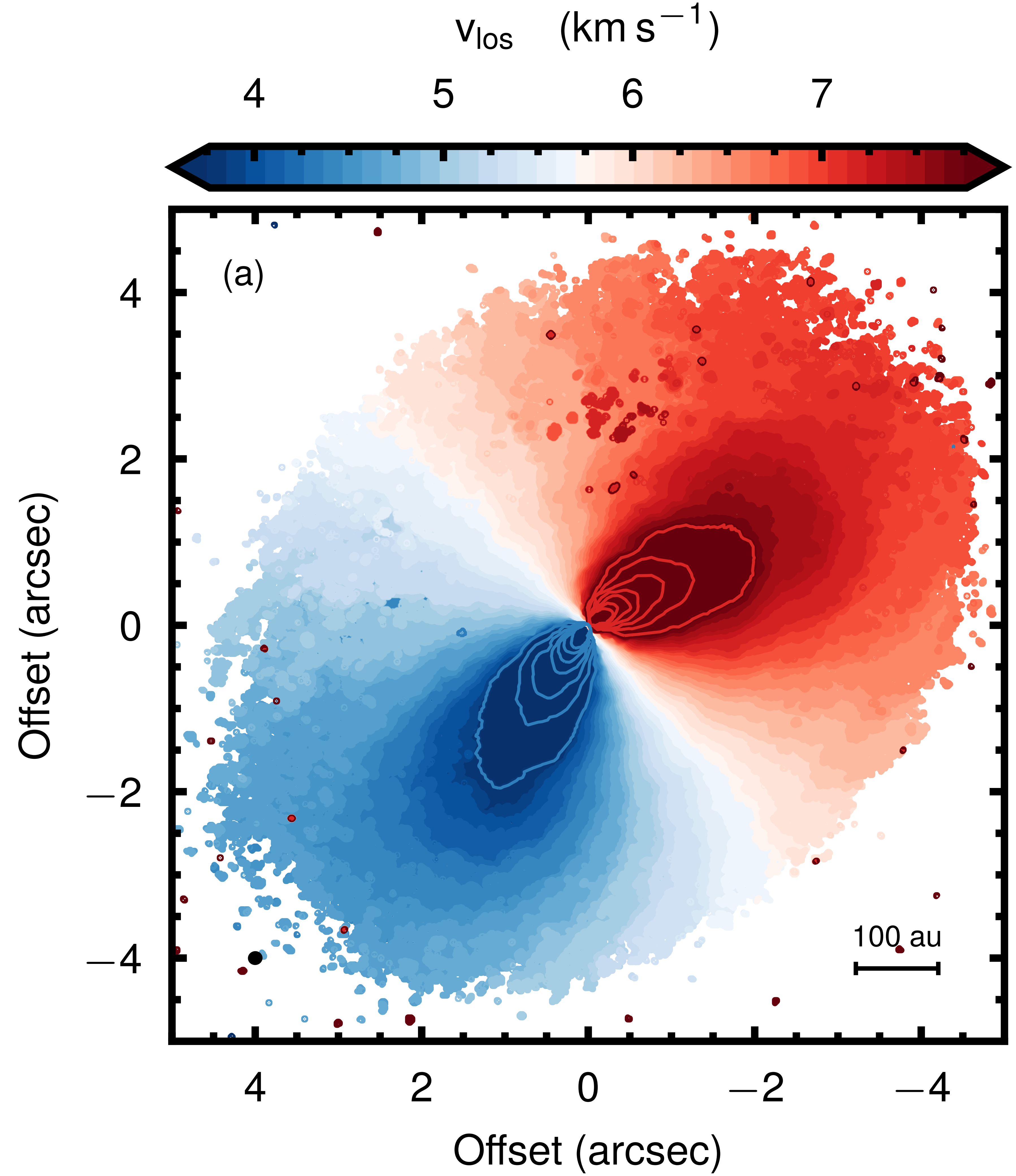
Wölfer+subm

It probes the disk *gas* dynamics, a fundamental driver regulating disk evolution and dust dynamics on small scales (e.g., turbulence) and the whole disk extent (e.g., winds).

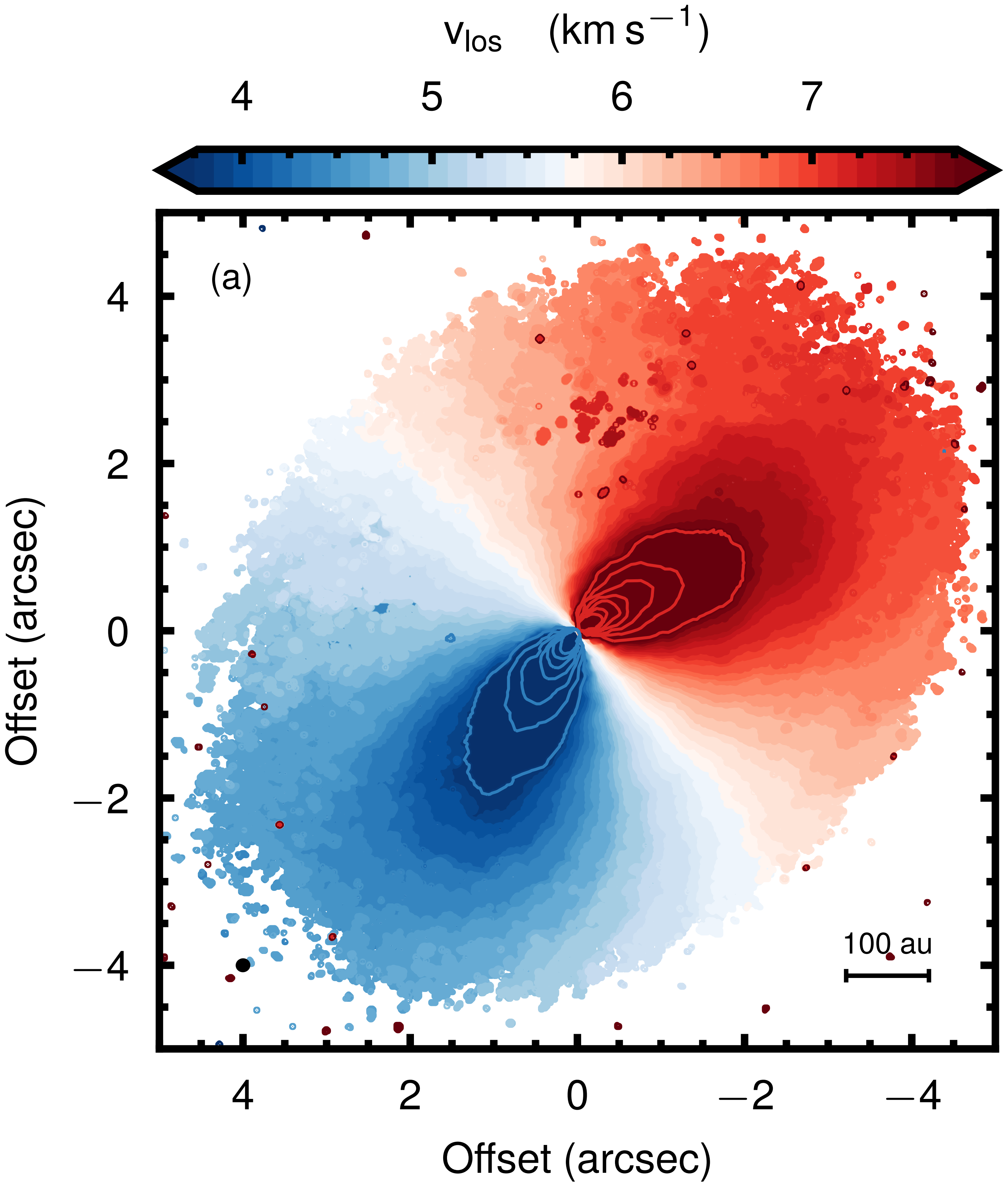
The dynamics can be used to trace ongoing physical processes, such as (M)HD-instabilities, planet-disk interactions, stellar fly-bys, etc.

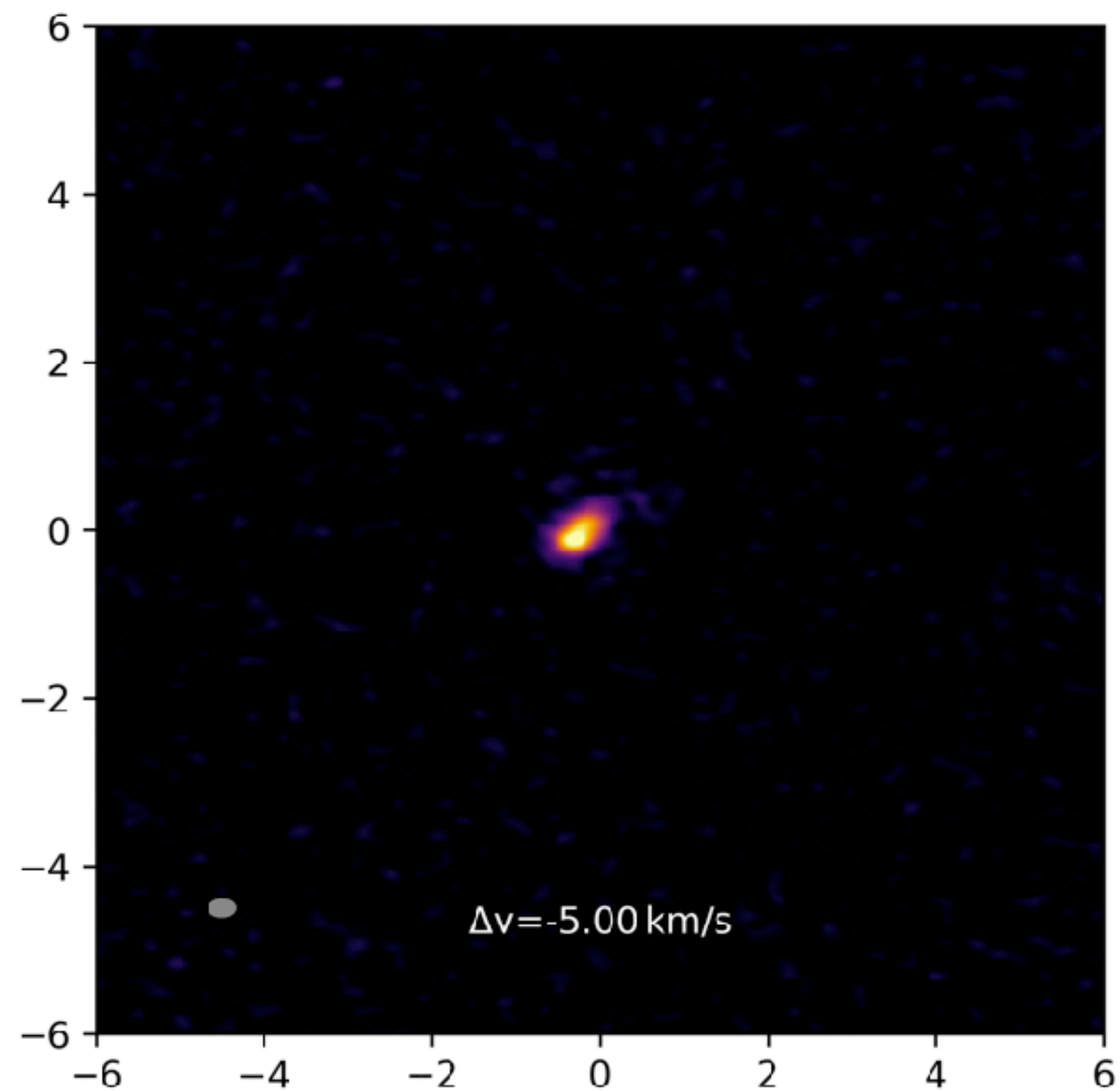

Disk kinematics
Keplerian rotation
Winds
Late
stage
infall
Meridional
flows
Spiral
wakes
planet
Upper CO layer
Turbulence
planet
+ CPD
Gap
Inclined and
eccentric
companion
Radial flows
Lower CO layer
Pinte+PPVII


Isella+2016

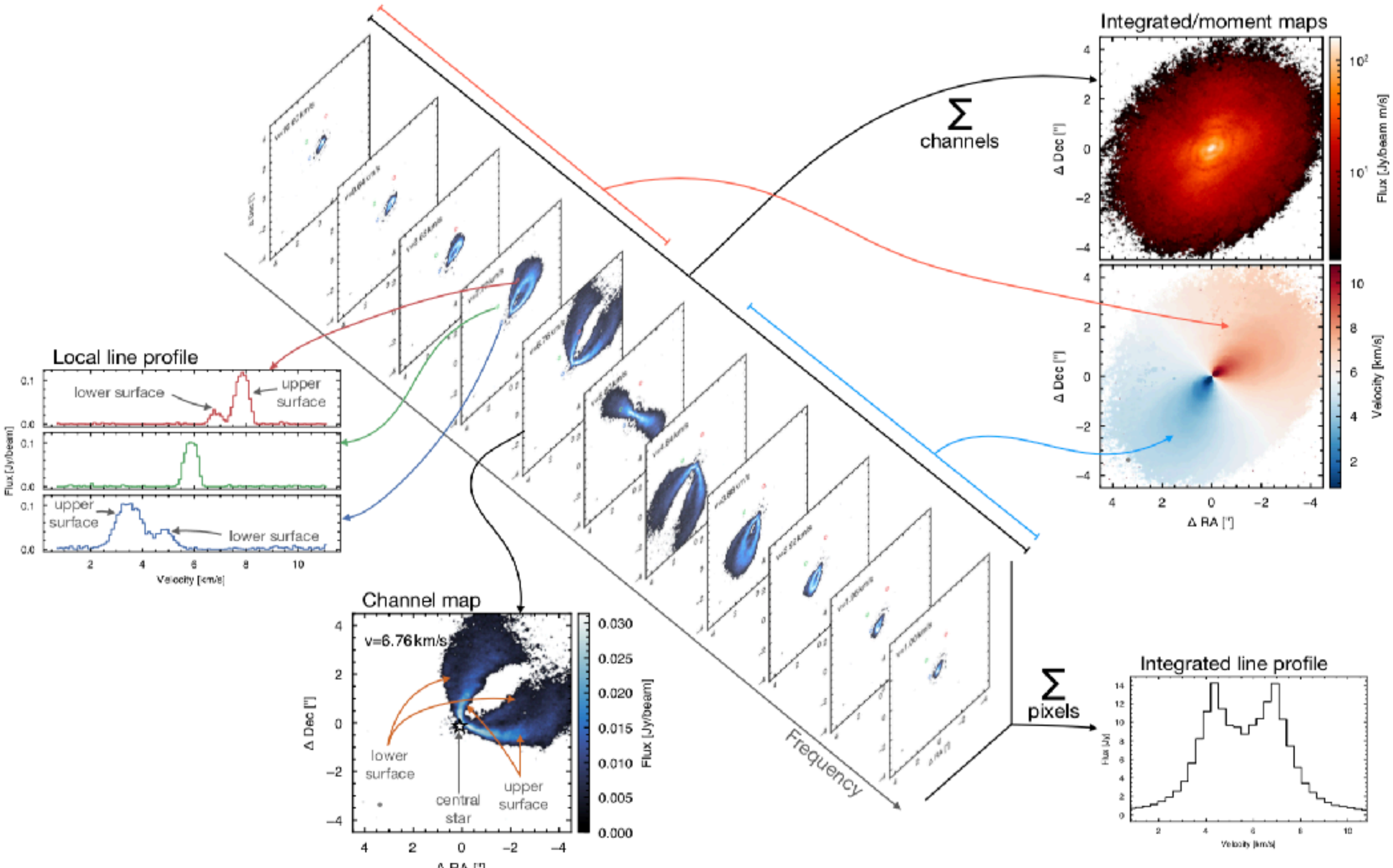

v_{los} (km s^{-1})
4
5
6
7
(a)
Offset (arcsec)
4
2
0
−2
−4
100 au
Offset (arcsec)

Disk kinematics





A data cube



Pinte+PPVII

Typical velocities in disks

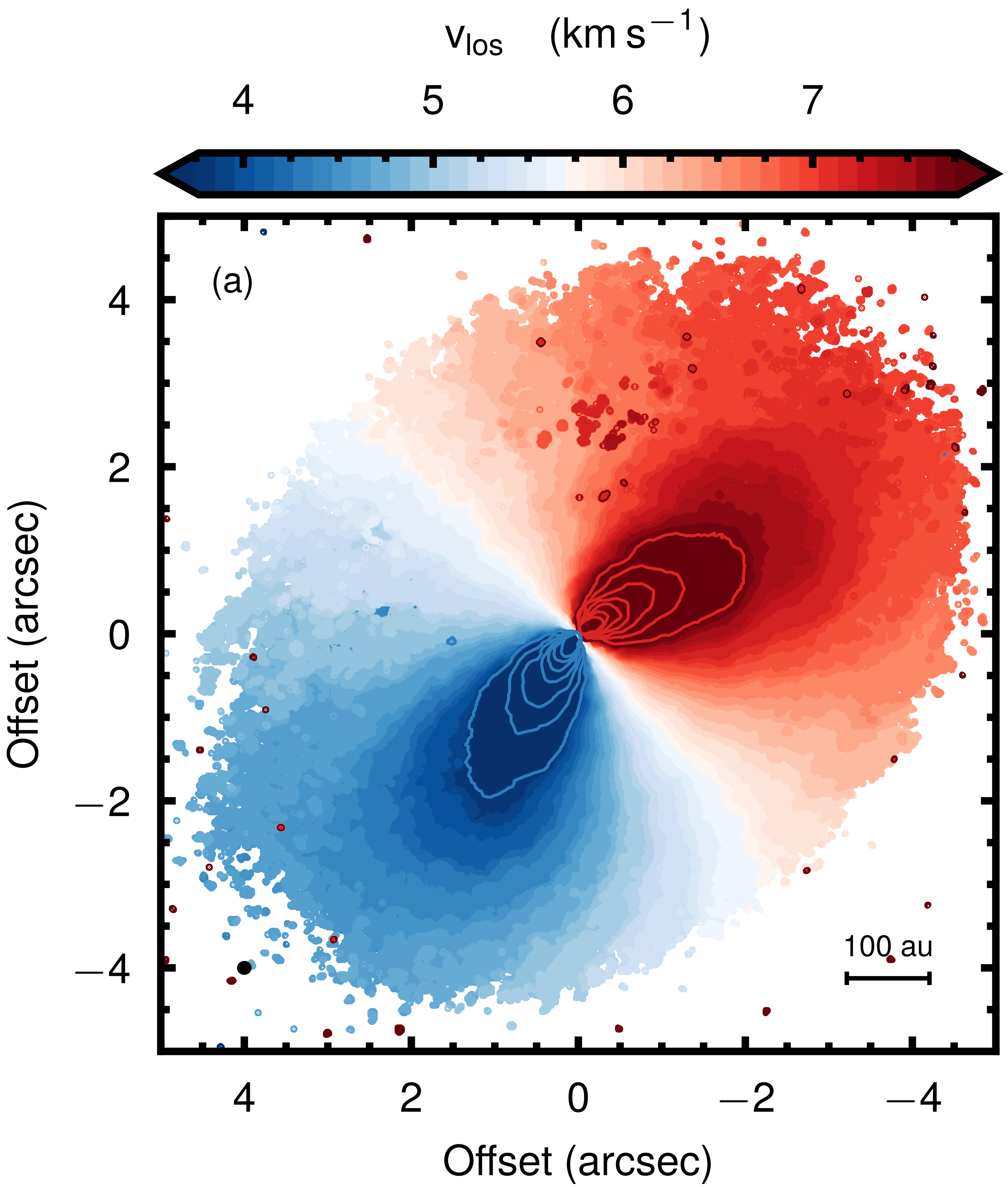
- Keplerian: $$v_{\phi,\mathrm{K}}(r) = \sqrt{\frac{GM_*}{r}} \approx 3.0\sqrt{\frac{M_*}{M_\odot}}\sqrt{\frac{100\mathrm{au}}{r}}\mathrm{km\,s^{-1}}$$
- Sound speed: $$c_\mathrm{s} = \sqrt{\frac{k_\mathrm{B}T_\mathrm{gas}}{\mu m_\mathrm{h}}} \approx 300\sqrt{\frac{T_\mathrm{gas}}{25\mathrm{K}}}\,\mathrm{m\,s^{-1}}$$
- Line width (thermal+turbulent): $$\Delta v = \sqrt{\frac{2k_\mathrm{B}T_\mathrm{gas}}{m_\mathrm{mol}} + \delta v_\mathrm{turb}^2} \approx \sqrt{(120\,\mathrm{m\,s^{-1}})^2 + \delta v_\mathrm{turb}^2}$$
- Accretion: $$|v_\mathrm{r,acc}| \sim \alpha\left(\frac{h}{r}\right)^2 v_\mathrm{Kep} \lesssim 10^{-4}\,v_\mathrm{Kep}$$

Typical velocities in disks

- Keplerian: $$v_{\phi,\,\mathrm{K}}(r) = \sqrt{\frac{GM_*}{r}} \approx 3.0\sqrt{\frac{M_*}{M_\odot}}\sqrt{\frac{100\mathrm{au}}{r}}\,\mathrm{km\,s^{-1}}$$
- Sound speed: $$c_\mathrm{s} = \sqrt{\frac{k_\mathrm{B}T_\mathrm{gas}}{\mu m_\mathrm{h}}} \approx 300\,\sqrt{\frac{T_\mathrm{gas}}{25\mathrm{K}}}\,\mathrm{m\,s^{-1}}$$
- Line width (thermal+turbulent): $$\Delta v = \sqrt{\frac{2k_\mathrm{B}T_\mathrm{gas}}{m_\mathrm{mol}} + \delta v_\mathrm{turb}^2} \approx \sqrt{(120\,\mathrm{m\,s^{-1}})^2 + \delta v_\mathrm{turb}^2}$$
- Accretion: $$|v_\mathrm{r,acc}| \sim \alpha \left(\frac{h}{r}\right)^2 v_\mathrm{Kep} \lesssim 10^{-4}\,v_\mathrm{Kep}$$

ALMA can reach an rms of 5K over one 50 m/s channel at 0.1″ in ~10h

Molecular emission height



Retrieving the disk surface is key to interpret astrochemical models, and to correctly de-project the velocity structures

The isovelocity curves show a clear bending due to the thick disk structure: it is possible to retrieve disk surface, but not the best way

Molecular emission height

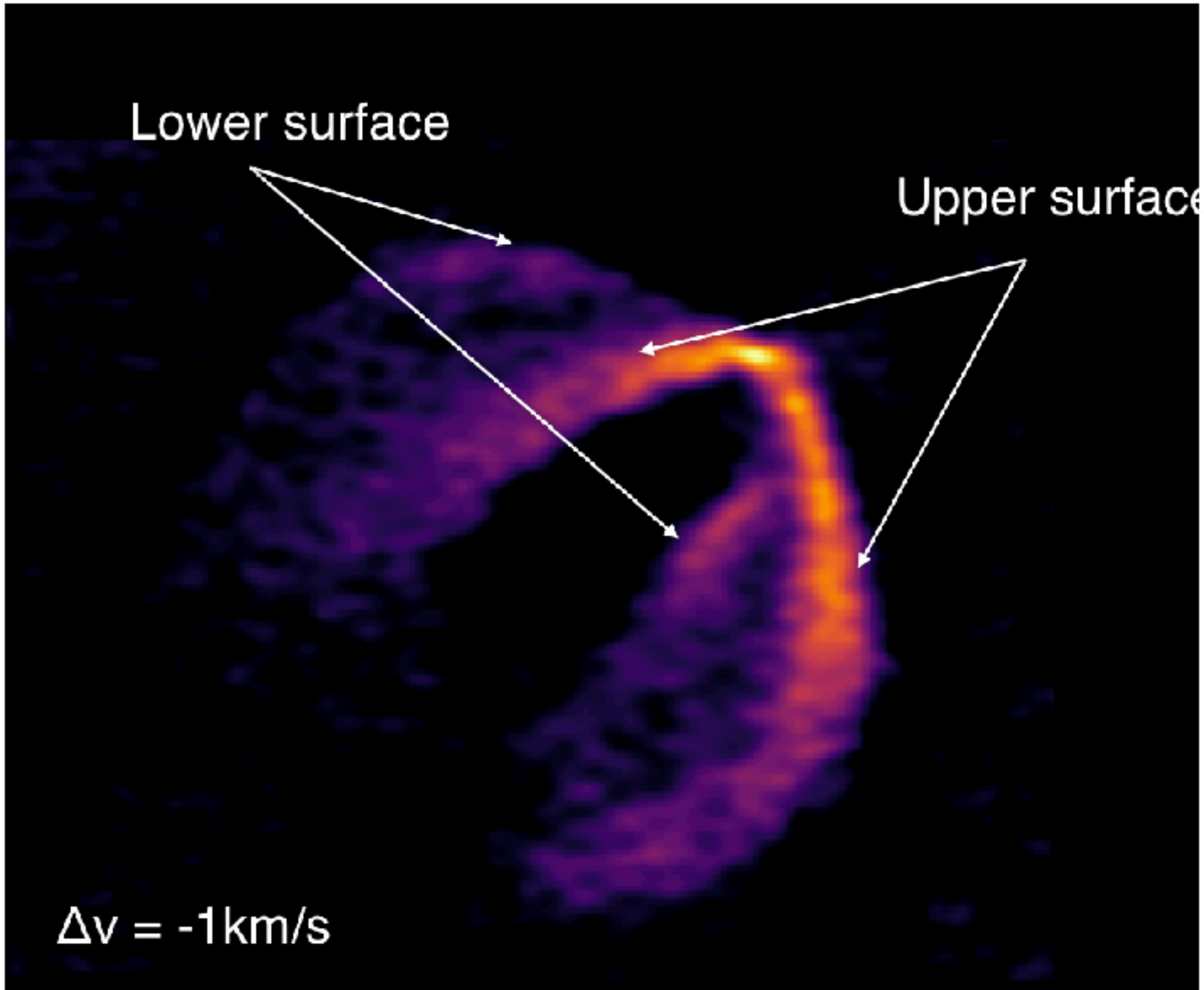
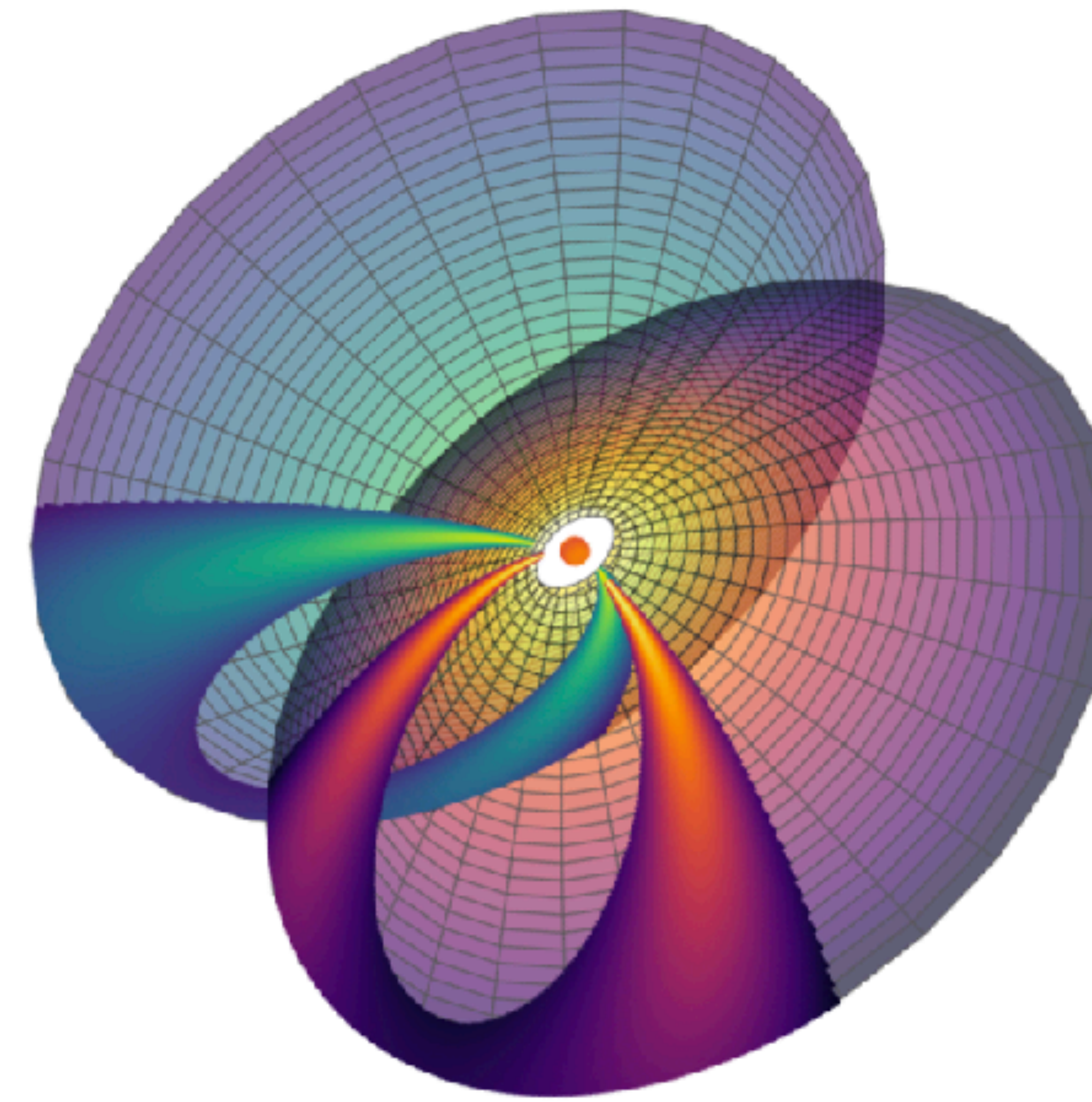
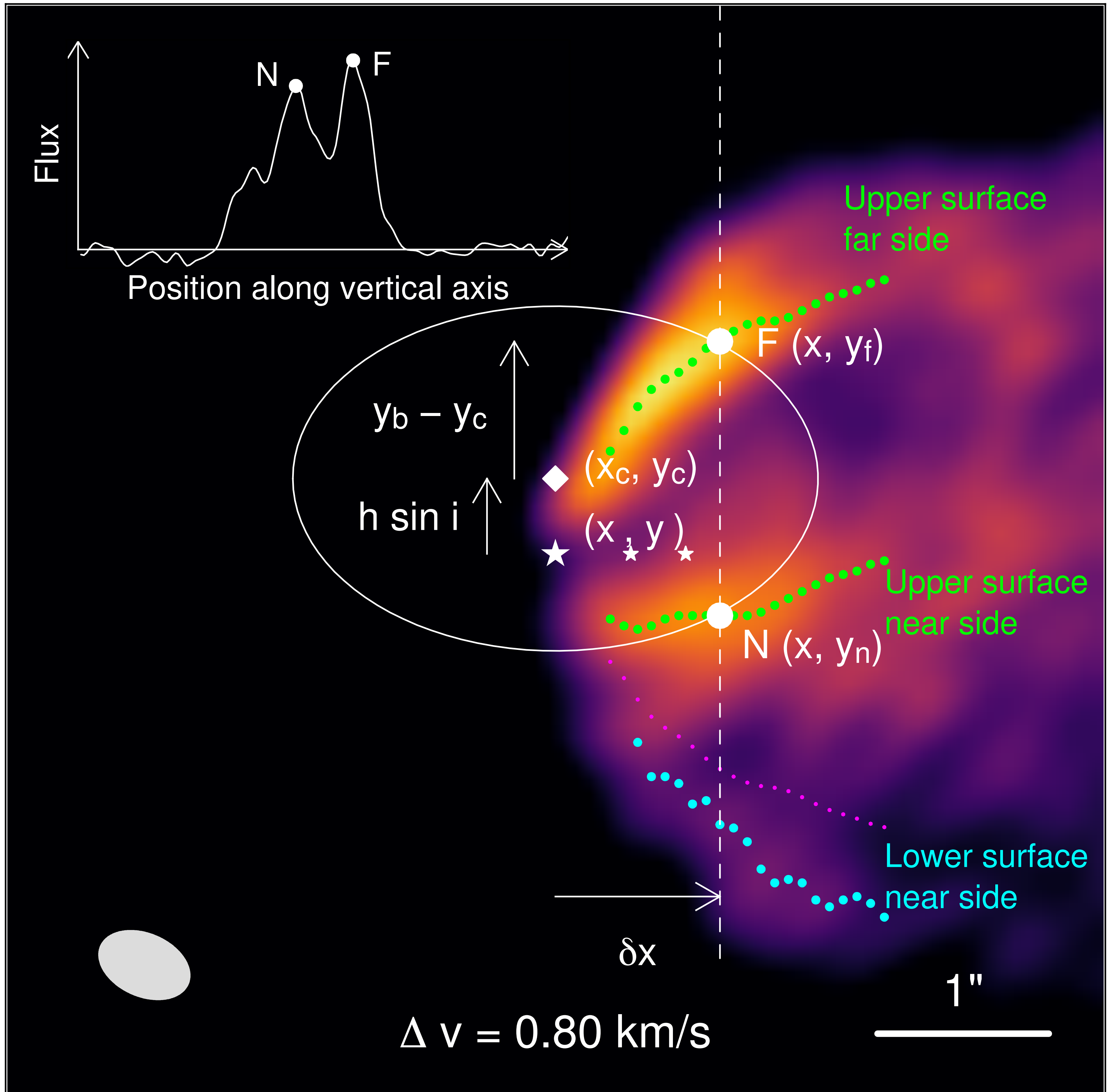


Image adapted from Pinte+2018b



Molecular emission height



Pinte+2018a

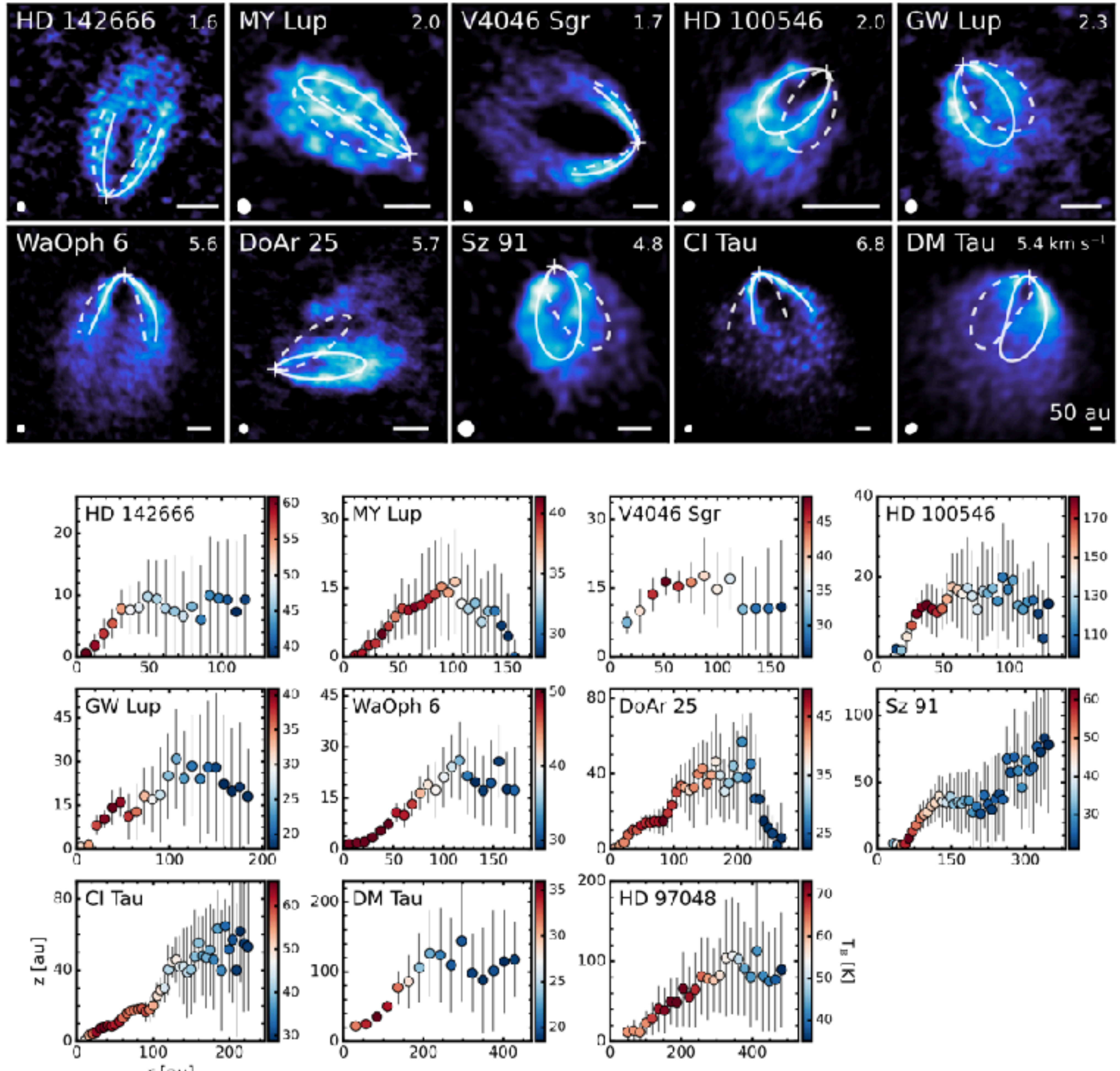
This allows to determine the emission of height of both the front and the back side of the disk, without fitting for a parametric model. Several packages are publicly available to do this for you

In general, we have that:

$$T_b = T_{\rm ex}(1 - e^{-\tau})$$

For optically thick lines, $\tau >> 1$. For lines in LTE, $T_{ex} = T_{kin}$.

We can use multiple lines to trace the disk thermal structure!


HD 142666 1.6
MY Lup 2.0
V4046 Sgr 1.7
HD 100546 2.0
GW Lup 2.3
WaOph 6 5.6
DoAr 25 5.7
Sz 91 4.8
CI Tau 6.8
DM Tau 5.4 km s⁻¹
50 au
HD 142666
MY Lup
V4046 Sgr
HD 100546
GW Lup
WaOph 6
DoAr 25
Sz 91
CI Tau
DM Tau
HD 97048
z [au]
r [au]
T_B [K]

Law+2022