1st dustbusters school on protostellar discs and planet formation

Hydrodynamic simulations of "planet-forming" discs

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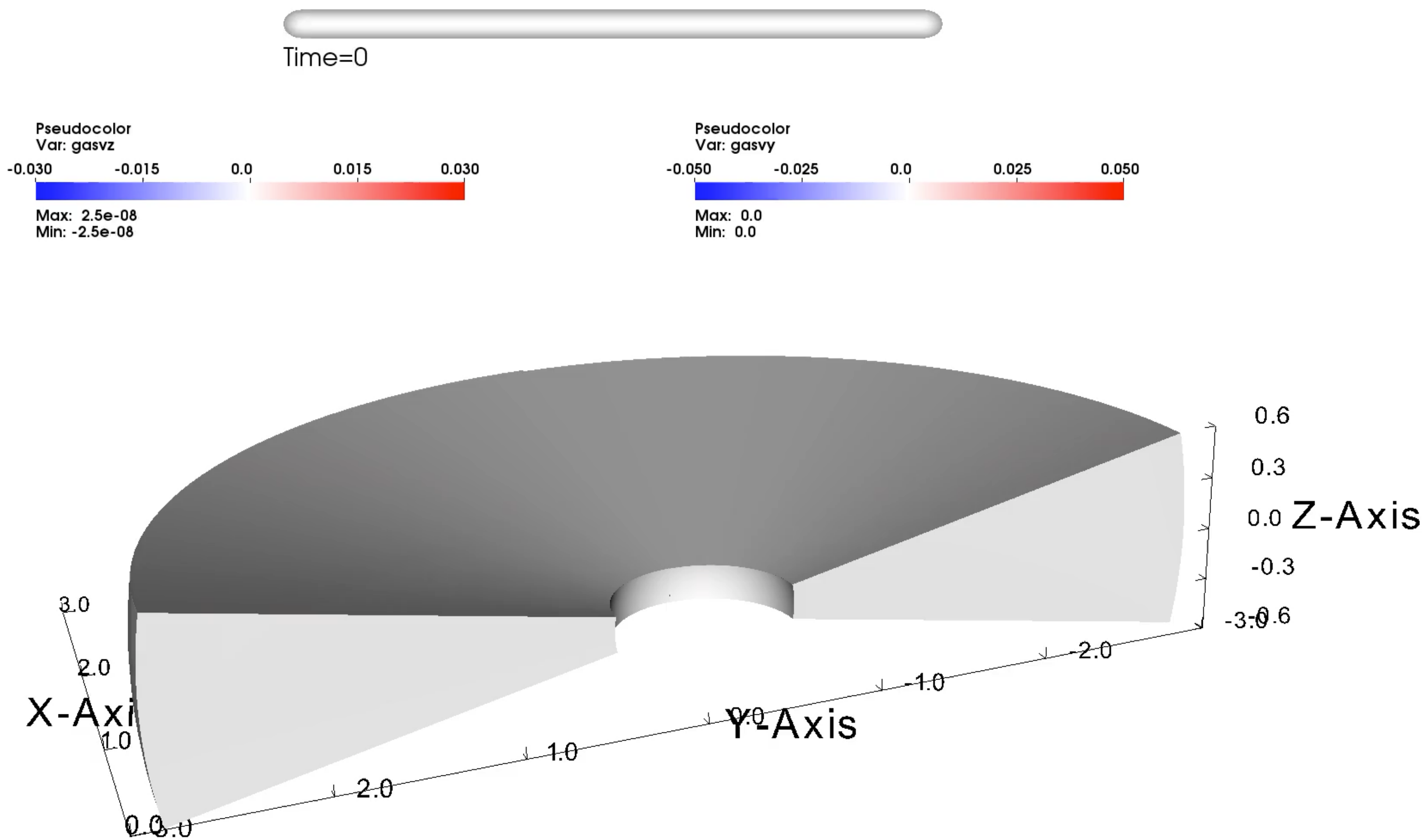
When/why do we need computer simulations?

- When a problem becomes too complicated to solve with pen and paper.
  - large number of bodies (e.g., n-body simulations where $n \gg 1$)
  - non-linear phenomena (e.g., HD/MHD instabilities)

- In Astronomy, lab experiments are impossible most of the time.
  - What would you do when you'd like to make your own planet?

One thing to keep in mind

- Computers do (and only do) what they are asked to do.
  - If you give your computer incorrect initial conditions, equations, assumptions, etc., it will not correct them for you!



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Pseudocolor
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-0.015
0.0
0.015
0.030
Max: 2.5e-08
Min: -2.5e-08
Pseudocolor
Var: gasvy
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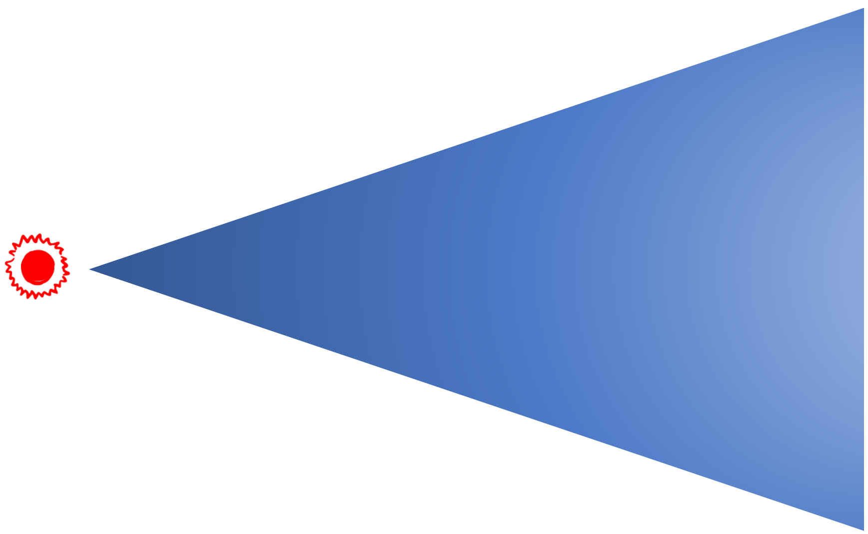
-0.025
0.0
0.025
0.050
Max: 0.0
Min: 0.0
0.6
0.3
0.0
Z-Axis
-0.3
-0.6
3.0
2.0
1.0
0.0
X-Axis
Y-Axis
2.0
1.0
-1.0
-2.0
-3.0

In this lecture, we will talk about

- Disc structure (initial conditions for any disc simulations)
- Gas-dust interaction
- Project introduction

Let's say we'd like to run a HD disc simulation with a star & a disk. What disc properties should we give to our computer?



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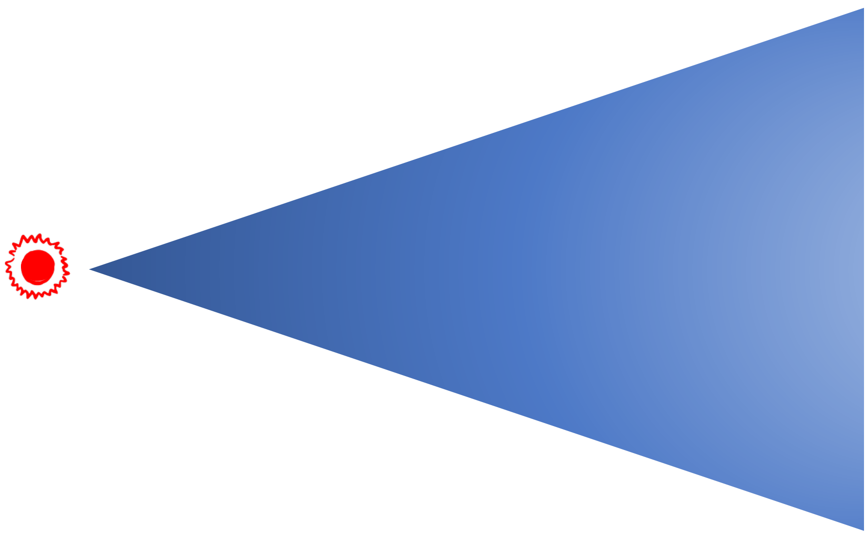

size, mass (or density), velocities, temperature, equation of state, heating/cooling, viscosity, …

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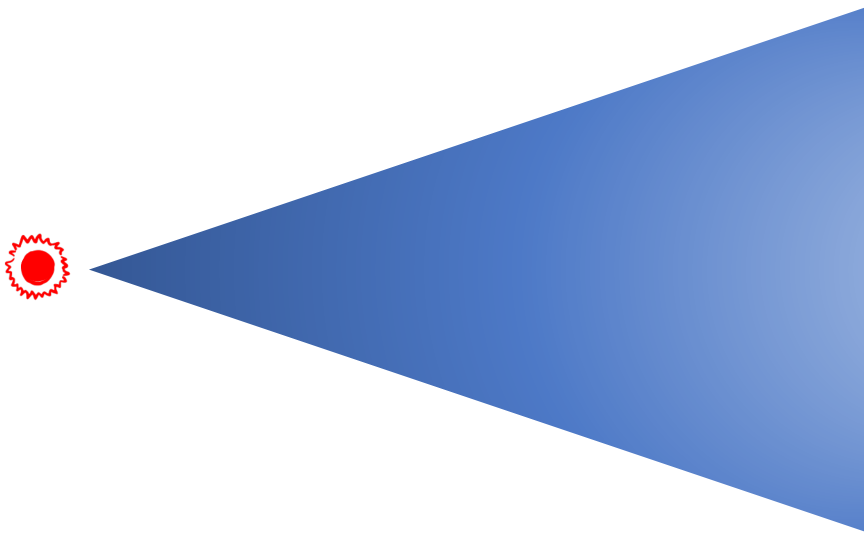
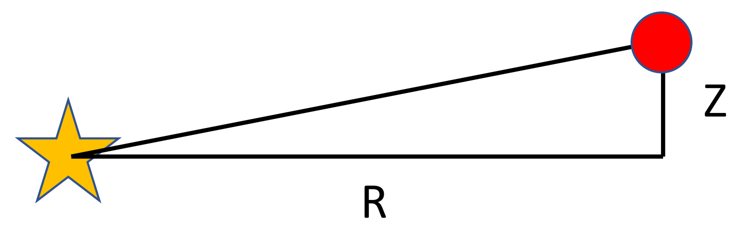


size, **mass (or density), velocities, temperature,** equation of state, heating/cooling, viscosity, …

Let’s consider a disk in hydrostatic equilibrium.

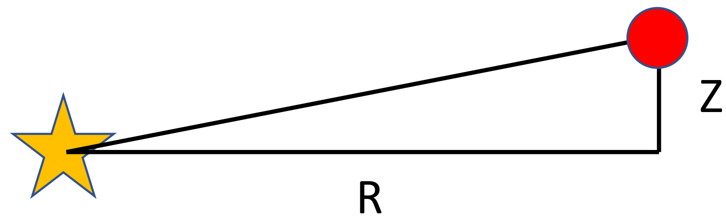
- Along each direction, all the relevant forces should be in balance.
  - Otherwise, you will see the disk evolves to find an equilibrium configuration (or even the simulation crashes)!

Force balance in the vertical direction



Force balance in the vertical direction

$$\begin{aligned}\frac{dP}{dZ} &= -\rho g_Z \\ &= -\rho \frac{GM_*}{(R^2+Z^2)} \frac{Z}{(R^2+Z^2)^{1/2}} \\ &= -\rho \frac{GM_*}{(R^2+Z^2)^{3/2}} Z\end{aligned}$$



Force balance in the vertical direction

$$\frac{dP}{dZ} = -\rho \frac{GM_*}{(R^2+Z^2)^{3/2}} Z$$

- In a vertically-isothermal disc adopting an isothermal EOS ($P = \rho c_s^2$),

$$\begin{aligned}
c_s^2 \frac{d\rho}{dZ} &= -\rho \frac{GM_*}{(R^2+Z^2)^{3/2}} Z \\
\frac{1}{\rho} d\rho &= -\frac{GM_*}{c_s^2} \frac{Z}{(R^2+Z^2)^{3/2}} dZ \\
\log[\rho(Z)/\rho_{\rm mid}] &= \frac{GM_*}{c_s^2} \left[\frac{1}{(R^2+Z^2)^{1/2}} \right]_0^Z \\
&= \frac{GM_*}{c_s^2} \left[\frac{1}{(R^2+Z^2)^{1/2}} - \frac{1}{R} \right]
\end{aligned}$$

Force balance in the vertical direction

$$\rho(R, Z) \quad = \quad \rho_{\rm mid} \exp\left(\frac{GM_*}{c_s^2} \left[\frac{1}{(R^2 + Z^2)^{1/2}} - \frac{1}{R} \right] \right)$$

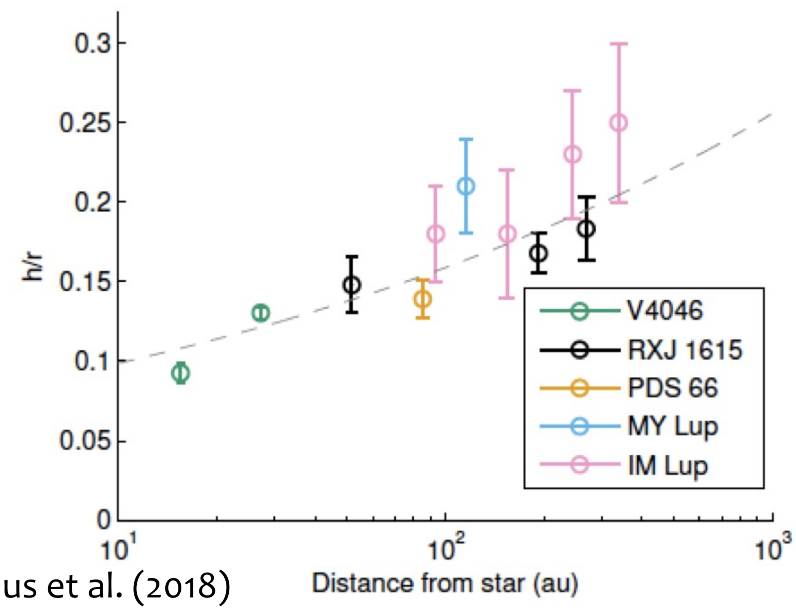
Force balance in the vertical direction

$$\begin{aligned}
\rho(R,Z) &= \rho_{\rm mid}\exp\left(\frac{GM_*}{c_s^2}\left[\frac{1}{(R^2+Z^2)^{1/2}}-\frac{1}{R}\right]\right) \\
&\simeq \rho_{\rm mid}\exp\left[-\frac{GM_*}{c_s^2}\frac{Z^2}{2R^3}\right],\ \text{when } R \gg Z \\
&= \rho_{\rm mid}\exp\left[-\Omega_K^2 Z^2/(2c_s^2)\right] \\
&= \rho_{\rm mid}\exp\left[-Z^2/2H^2\right],\ \text{where } H \equiv c_s/\Omega_K
\end{aligned}$$

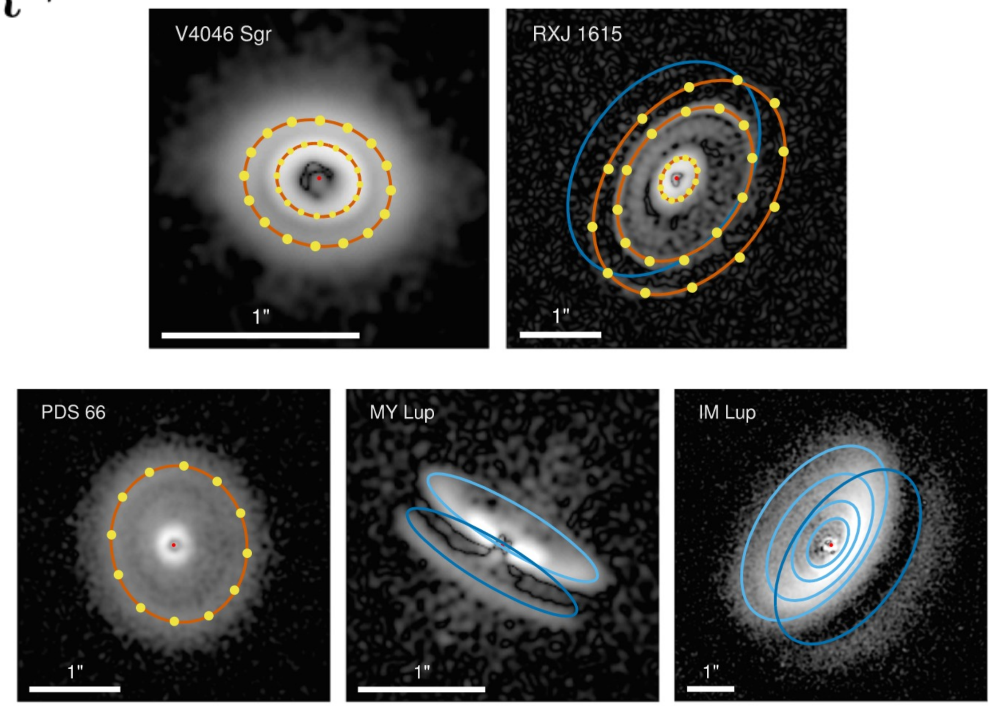
Disc aspect ratio: $\dfrac{H}{R} = \dfrac{c_s}{R\Omega_K} = \dfrac{c_s}{v_K}$

Protoplanetary discs are "flared"

$$\frac{H}{R} = \frac{c_s}{v_K} \propto \frac{T^{1/2}}{R^{-1/2}} \propto \frac{R^{-1/4}}{R^{-1/2}} \propto R^{1/4}$$

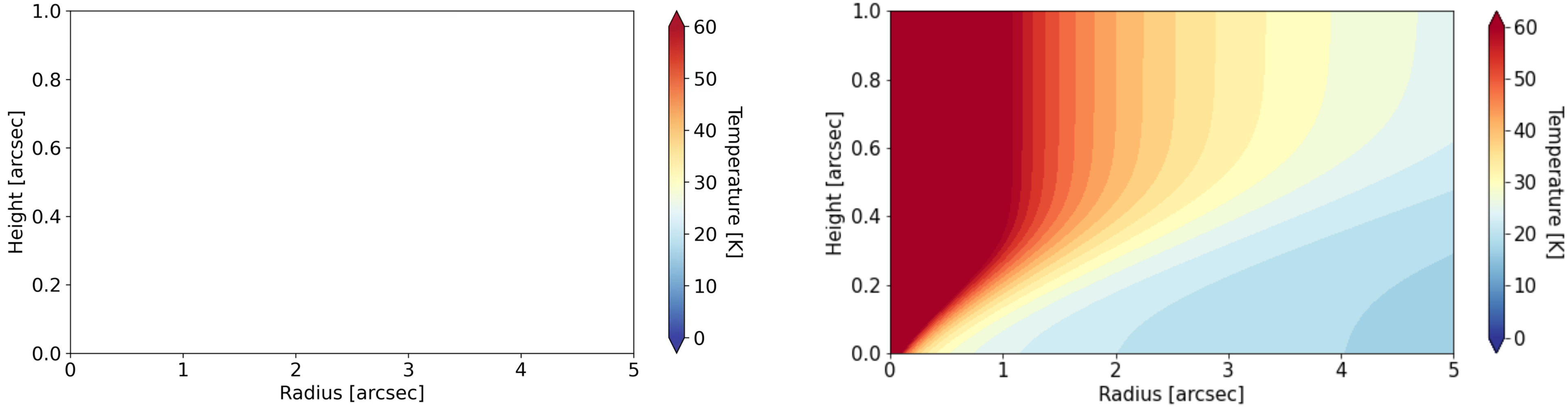


Avenhaus et al. (2018)



What if the disc temperature is vertically stratified?

- HD 163296 temperature using CO isotopologues



Law et al. (2021)

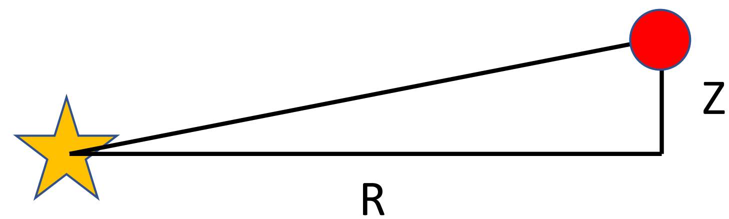
What if the disc temperature is vertically stratified?

$$\frac{dP}{dZ} = -\rho \frac{GM_*}{(R^2 + Z^2)^{3/2}} Z$$

$$\frac{d}{dZ}(\rho c_s^2) = -\rho \frac{GM_*}{(R^2 + Z^2)^{3/2}} Z$$

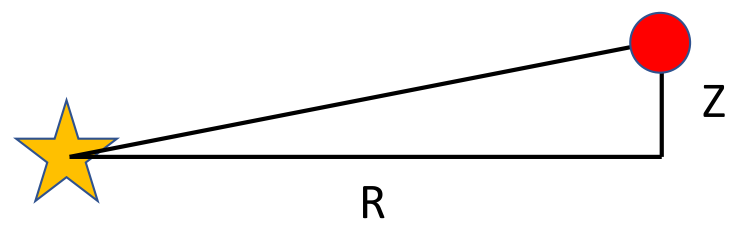
$$\rho_{\rm g}(Z) = \rho_{\rm g,mid} \frac{c_{s,\rm mid}^2}{c_s^2(Z)} \exp\left[-\int_0^Z \frac{1}{c_s^2(Z')} \frac{GM_* Z'}{(R^2 + Z'^2)^{3/2}} dZ' \right]$$

Force balance in the radial direction



Force balance in the radial direction

$$\frac{v_\phi^2}{R} = \frac{GM_*R}{(R^2+Z^2)^{3/2}} + \frac{1}{\rho}\frac{dP}{dR}$$



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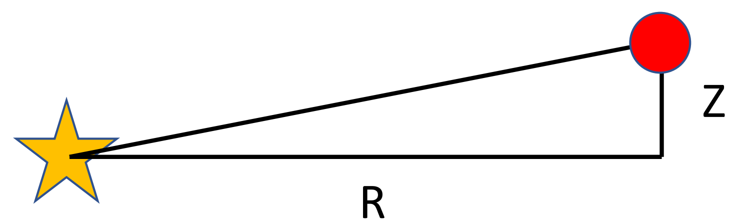

$$T(R) = T_0 \left(\frac{R}{R_0}\right)^q$$

$$\rho(R,Z) \quad = \quad \rho_{\rm mid} \exp\left(\frac{GM_*}{c_s^2}\left[\frac{1}{(R^2+Z^2)^{1/2}} - \frac{1}{R}\right]\right)$$

$$\rho_{\rm mid}(R) = \rho_0 \left(\frac{R}{R_0}\right)^p$$

$$\Omega(R,Z) = \Omega_K \left[(p+q)\left(\frac{H}{R}\right)^2 + (1+q) - \frac{qR}{\sqrt{R^2+Z^2}}\right]^{1/2}$$

The equilibrium rotational velocity has vertical shear.

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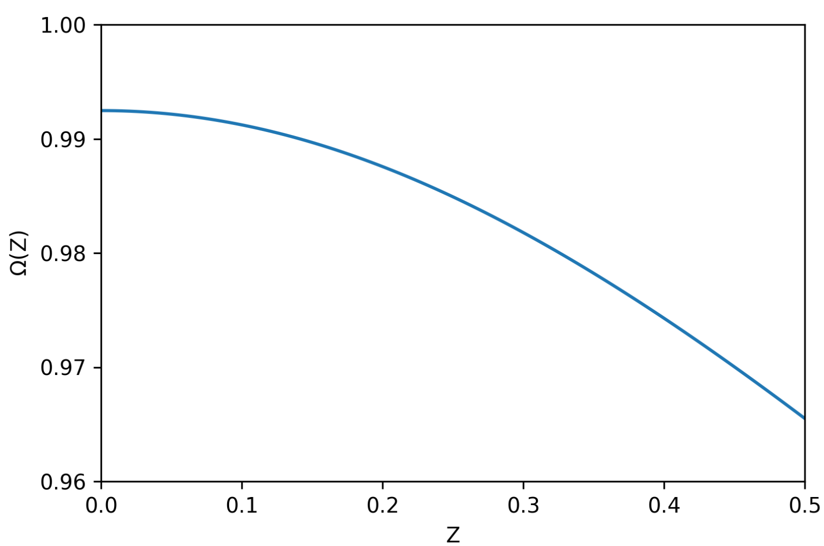

At R=1, adopting p=-1 & q=-0.5

The vertical shear in the rotational velocity of the disc can trigger an instability: VSI

In summary, if you'd like to run a (M)HD simulation

1. Define the temperature structure T(R,Z)

2. Define the density structure

$$\rho(R,Z) = \rho_{\rm mid} \exp\left(\frac{GM_*}{c_s^2}\left[\frac{1}{(R^2+Z^2)^{1/2}} - \frac{1}{R}\right]\right) \qquad \rho_{\rm mid}(R) = \rho_0 \left(\frac{R}{R_0}\right)^p$$

3. Define the velocity structure

$$\Omega(R,Z) = \Omega_K \left[(p+q)\left(\frac{H}{R}\right)^2 + (1+q) - \frac{qR}{\sqrt{R^2+Z^2}}\right]^{1/2}$$

$$v_R = v_Z = 0$$

What about dust?

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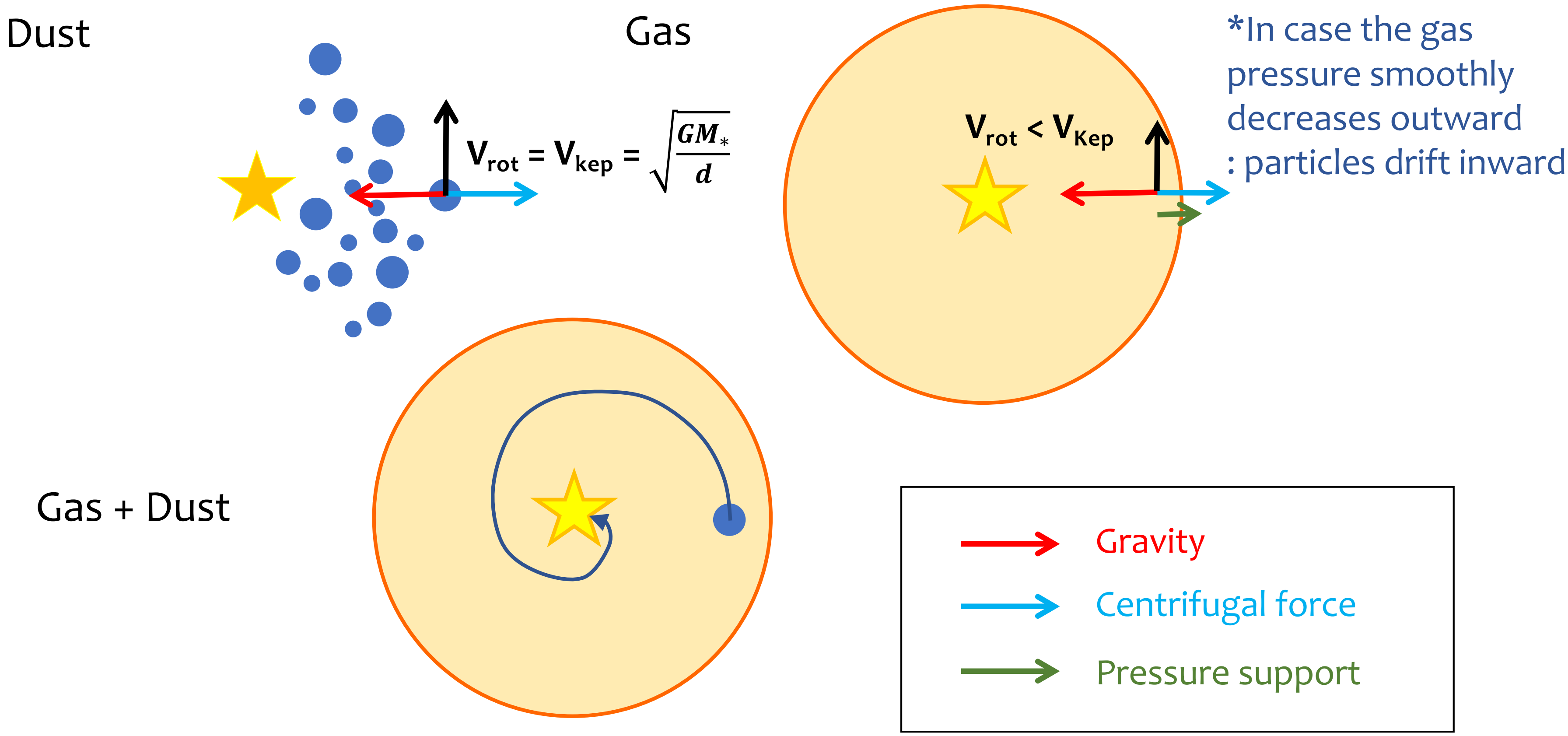

Aerodynamic drag

- Epstein drag: drag occurs as the particle collides with individual gas molecules.

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$$F_D = -\frac{4}{3}\pi a^2 \rho_g \Delta v v_{\mathrm{th}}$$

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- The stopping time is given by

$$\begin{aligned} t_{\rm stop} &= m\Delta v/|F_D| \\ &= \frac{\rho_s}{\rho_g}\frac{a}{v_{\rm th}} \end{aligned}$$

Stokes number

- Dimensionless stopping time $\mathrm{St} = t_{\mathrm{stop}}\Omega_K = \frac{\pi\rho_s a}{2\Sigma_g}$

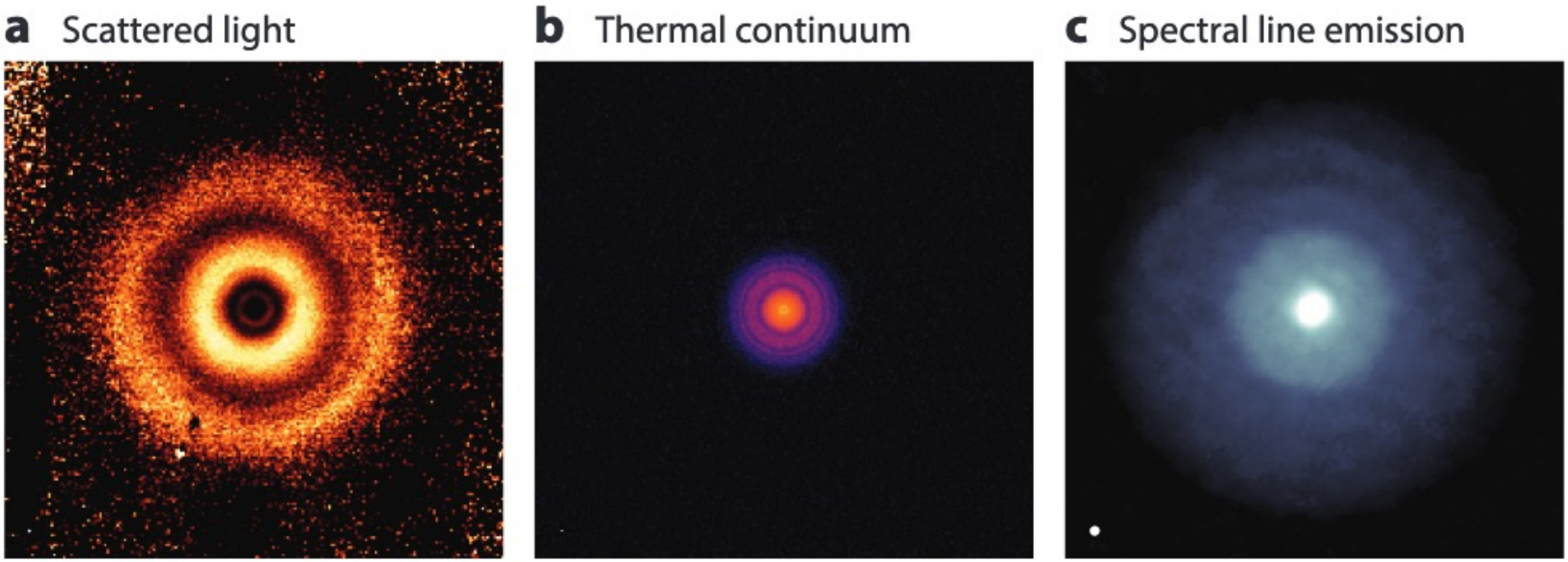
- When St << 1, particles follow the gas motion.
- When St >> 1, particles decouple from the gas.
- When St ~ 1, particles are marginally coupled to the gas.

Radial drift “problem” or meter-size “barrier”

- The timescale for the radial drift of St = 1 particles is only 1000 orbits.
- At 1 au from a solar-mass star, this is only 1000 years!

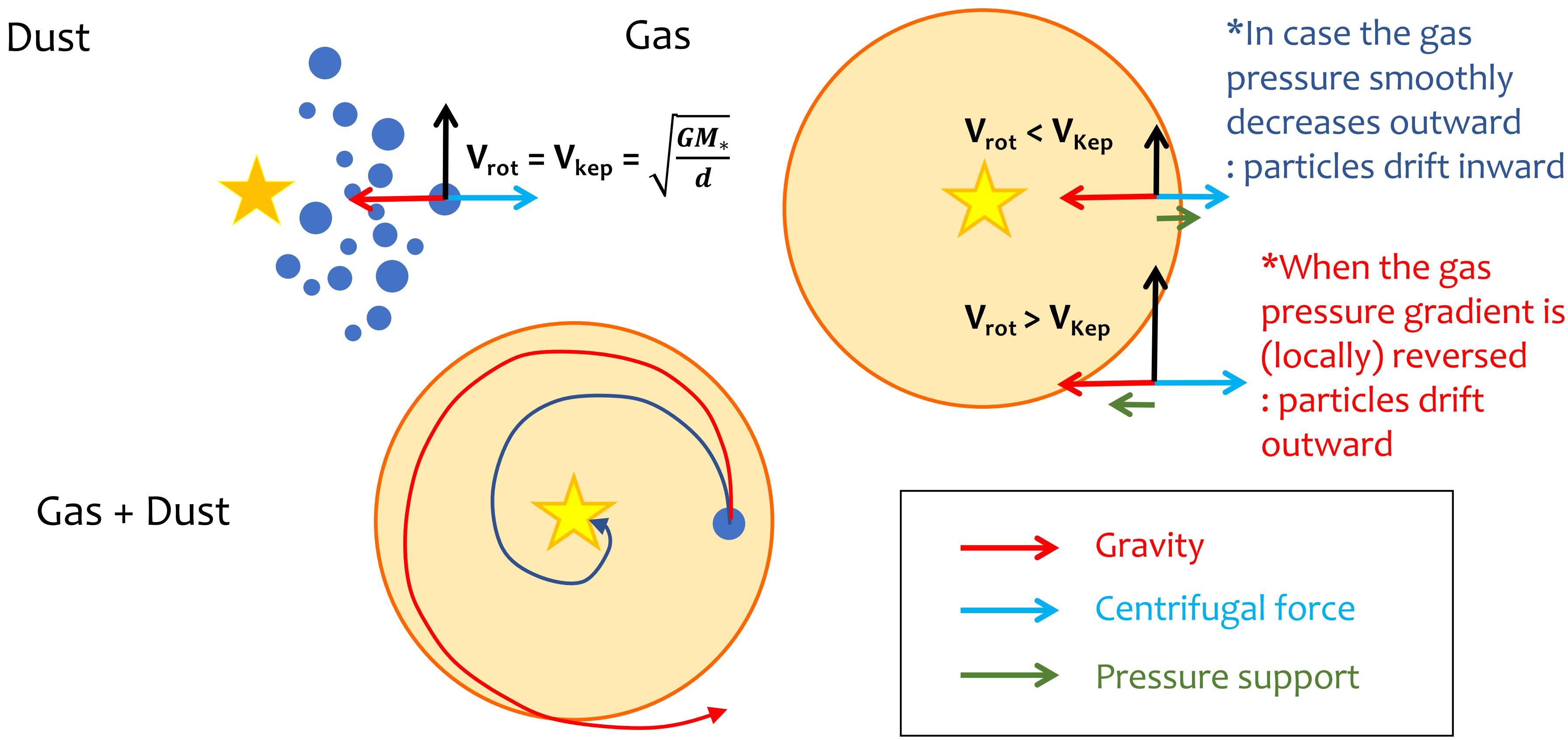
See e.g., Weidenschilling (1977)

Evidence of radial drift



Andrews (2020); TW Hya

Pressure bumps can trap dust.



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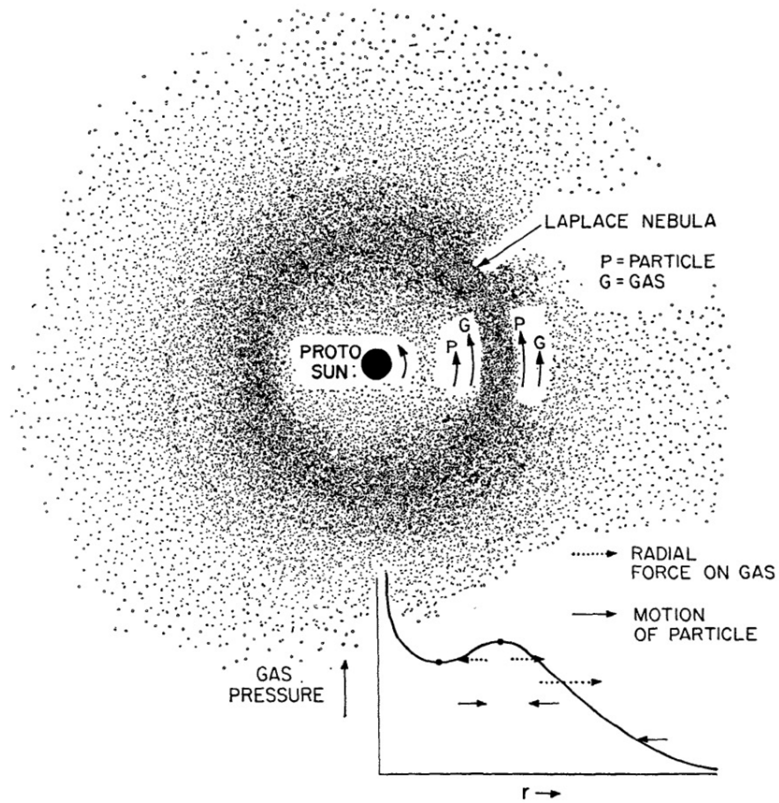
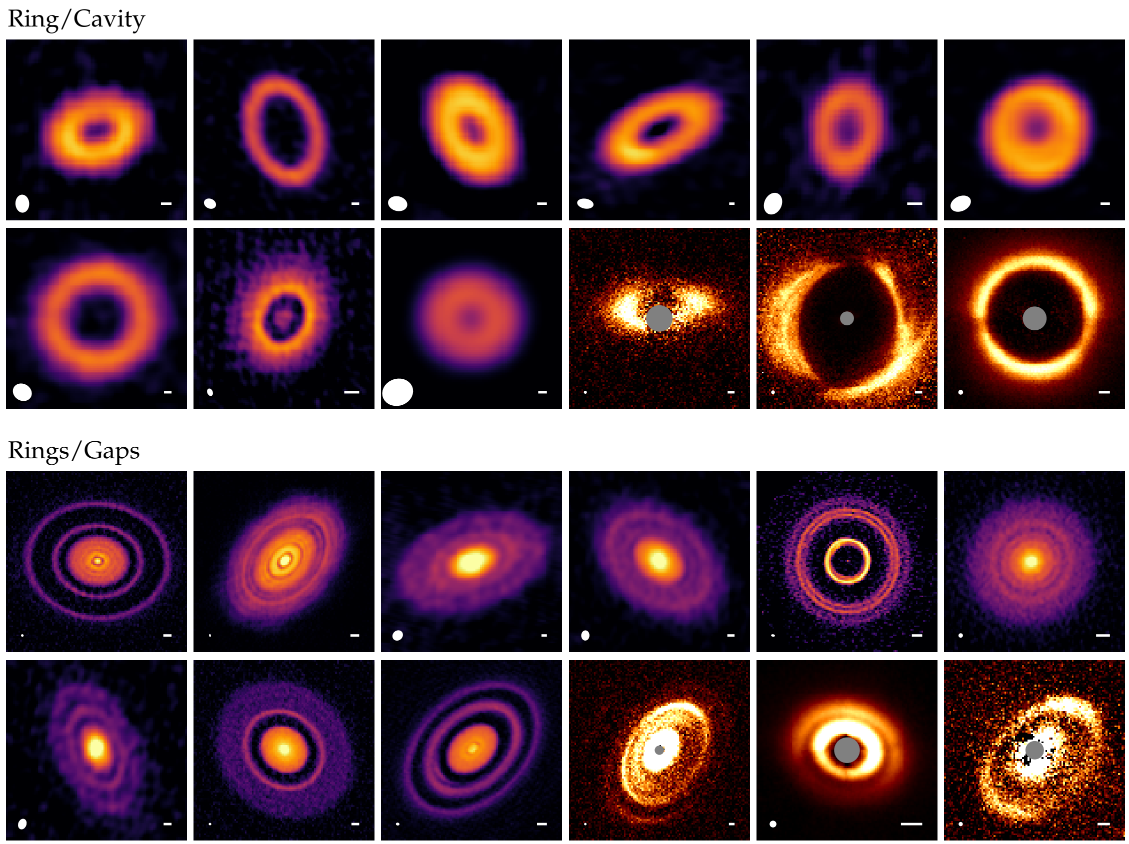


EFFECT OF GAS PRESSURE GRADIENT ON PARTICLE MOTION

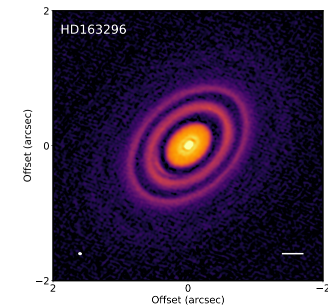
Whipple (1972)

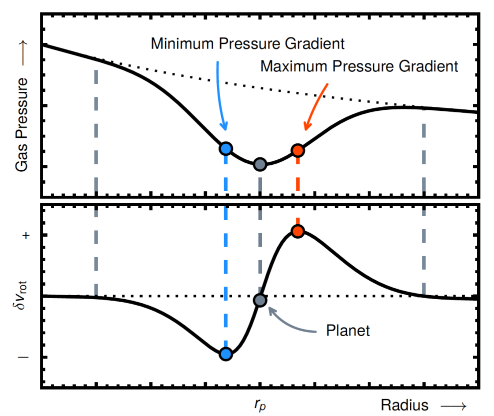
Pressure bumps can trap dust.

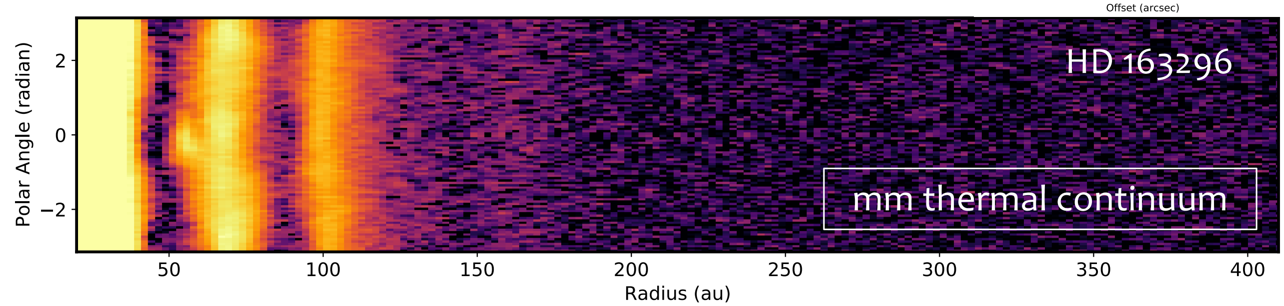


Andrews (2020)

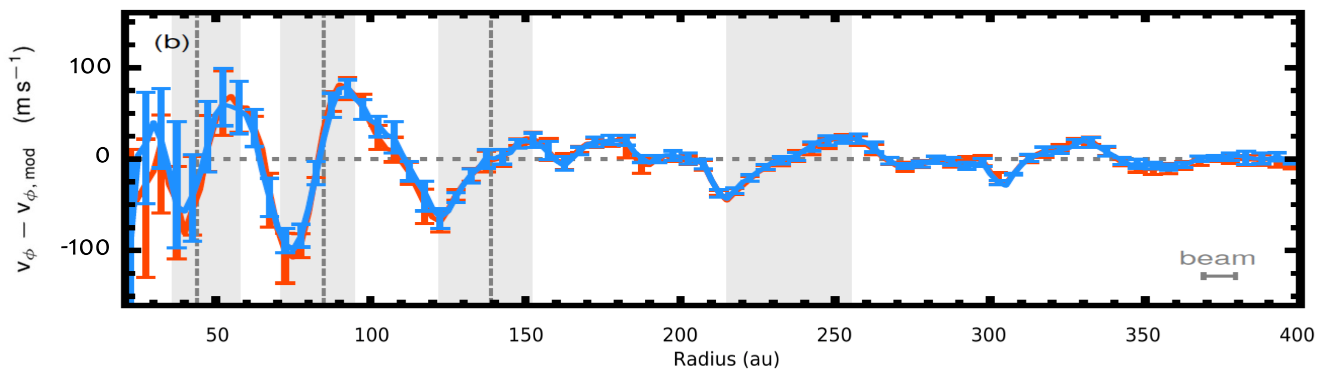
Evidence of dust trapping





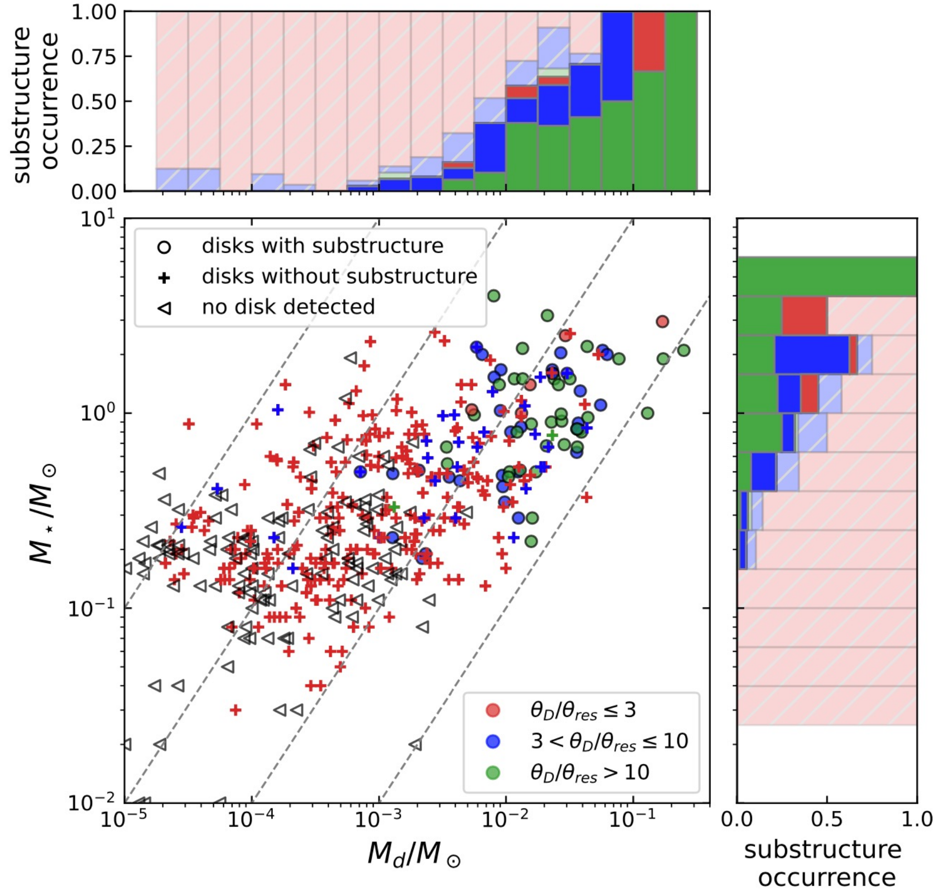


$$\frac{v_{\rm rot}^2}{r} = \frac{GM_\star r}{(r^2 + z^2)^{3/2}} + \frac{1}{\rho_{\rm gas}} \frac{\partial P}{\partial r}$$



Teague, Bae et al. (2018), Teague, Bae & Bergin (2019, Nature), see also Rosotti et al. (2020)

Dust trapping might be happening ubiquitously.



Bae et al. (PPVII)

Vertical settling

- When the thickness of a dust layer is determined by turbulent diffusion and vertical settling, $t_{diff} = t_{sett}$.

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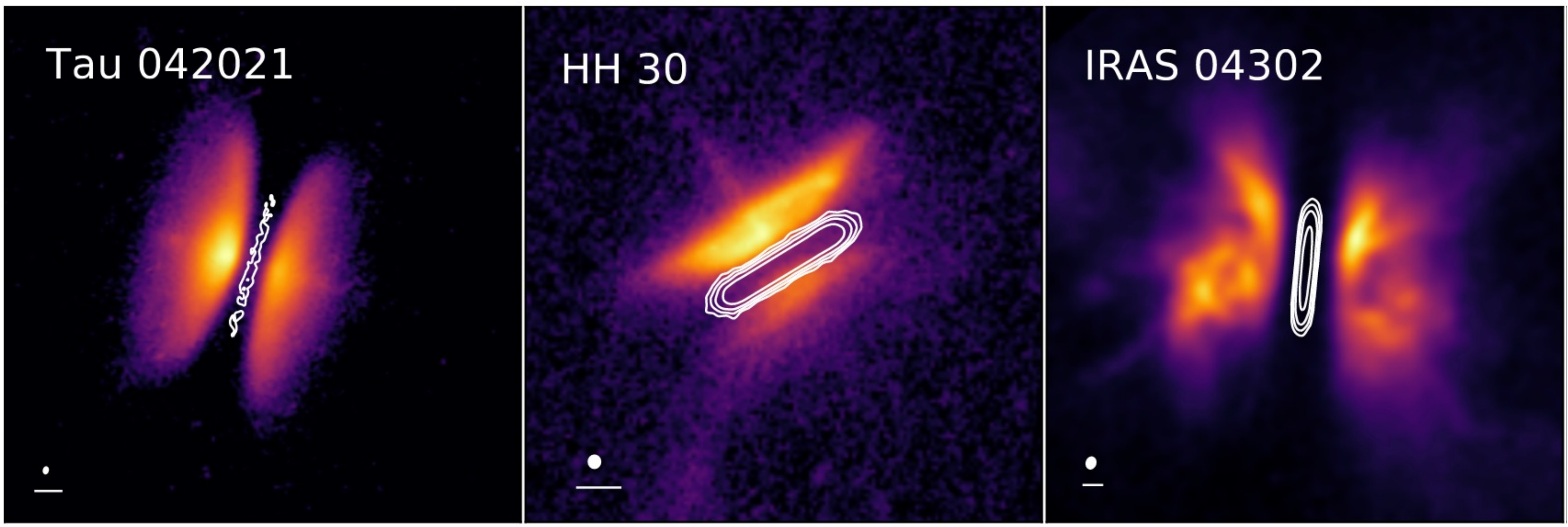
- $t_{diff} = H_d^2/D_Z$, where $D_Z = \alpha_Z c_s^2/\Omega_K$.
- $t_{sett} = 1/(St\ \Omega_K)$ from $F_{grav} = F_D$
- $H_d = H_g\sqrt{\alpha/St}$

Evidence of vertical settling



Color background: HST

White contours: ALMA continuum

Villenave et al. (2020)

In summary,

- the dust distribution can be very different from the gas distribution;
- the gas-dust interaction can be understood through aerodynamic drag;
- the level of aerodynamic drag depends on the Stokes number.

- The beauty is that we now “see” gas-dust interaction happening!

Few things to consider when you add dust in your simulation

- There’s not necessarily an “equilibrium” distribution.
  - When/where should I add dust?

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- There's not necessarily an "equilibrium" distribution.
  - When/where should I add dust?
- Dust can grow in size.
  - Solving full coagulation and fragmentation along with HD is computationally expensive (see e.g., Drazkowska et al. 2019).
  - A more affordable approach would be to use a limited number of dust populations (e.g., small vs. large) in an azimuthally/vertically-averaged setup (e.g., Brauer et al. 2008, Birnstiel 2010, 2012).

Projects

1. Planet-disk interaction and continuum observations
2. Vertical shear instability in molecular line observations

* FARGO3D/RADMC-3D setup files, output files, and Jupyter notebooks are available on this google drive.

* Jupyter notebooks are under the RADMC-3D folder of each project.